Objective

- Select a set of Representative Suite Common Form models
  - Represent a set of mutually exclusive and collective exhaustive models.

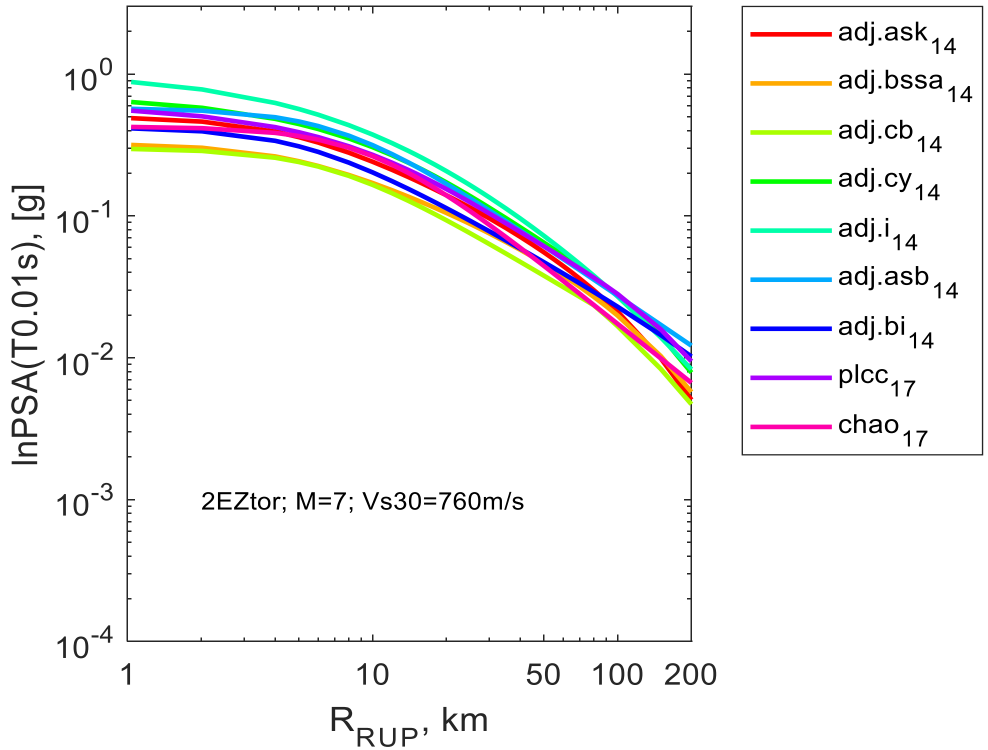
- Assign the weights to the Candidate GMPEs or Representative Suite GMPE Common Form models

- Select the Center Body and Range of Technically Defensible Interpretation (the CBR of the TDI) GMPEs for PSHA,

Select Candidate GMPEs for Hazard Calculation

- **Candidate GMPEs**
  R_{RUP} −based Models:
  - Adj.ASK14
  - Adj.CB14
  - Adj.CY14
  - Adj.I14
  - PLCC17
  - Chao17
- **Candidate GMPEs**
  R_{JB} −based Models
  - Adj.BSSA14
  - Adj.ASB14
  - Adj.Bi14

A total of 9 candidate GMPEs were selected



Approaches for developing continuous Distribution of the Median Prediction Using Sammon's Mapping

- Select Candidate GMPEs & Common Forms
- Refit the sampled candidate GMPEs using common forms
- Sample Synthetic GMPEs using the coefs.variance-covariance data
- Visualization GMPEs on the Sammon Map
- Identify the Center, Body and Range of GMPE Models on 2-D Sammon Map
- Select Representative common form models
- Weighting Computation using recorded data
  - > Residual weights (wR)
  - > Likelihood weights (wLL)
  - > Prior weights (wPri)
  - > Posterior weights (wPos)

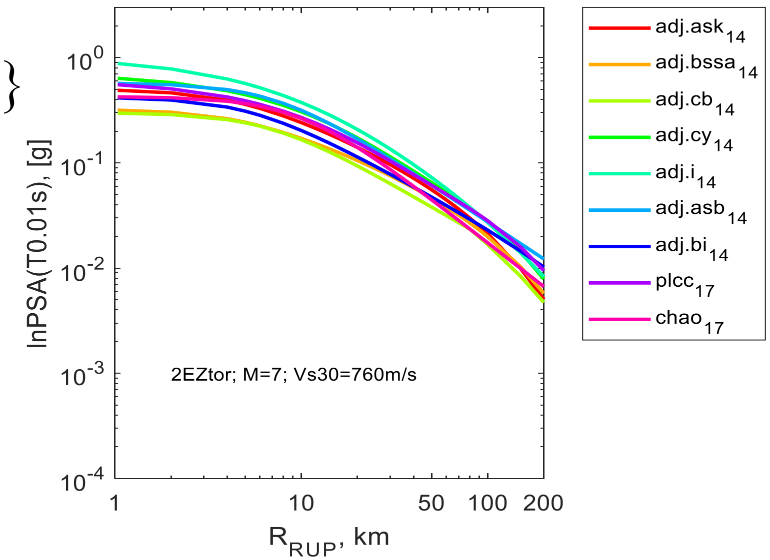
Euclidian Distance between GMPEs

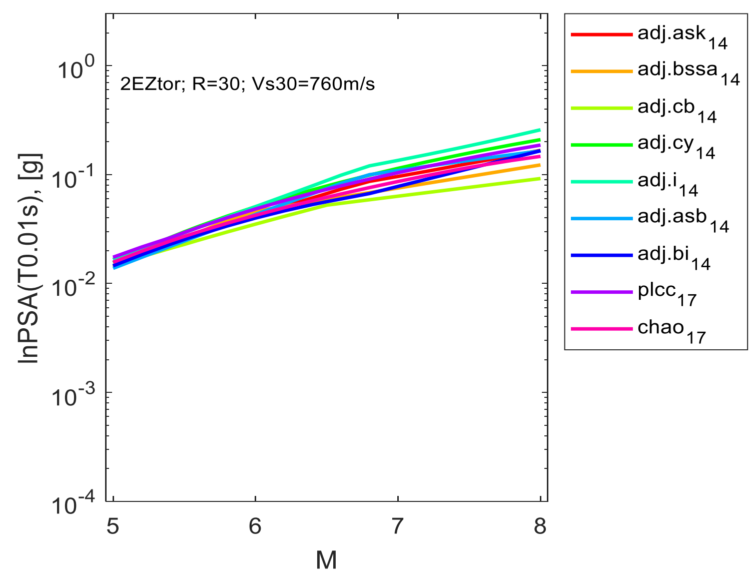
Sammon's map configuration:

- Given a set of scenarios $\{M, R_{Rup}, and\ Z_{TOR}\}$ ~Vector of ground motions
  - Euclidian distance between GMPEs
    - $\varepsilon_{ij} = \sqrt{\frac{1}{N}\sum_1^N (GMPE_i - GMPE_j)^2}$
  - Euclidian distance between GMPEs on 2-D map
    - $\delta_{ij}^{map} = \sqrt{\frac{1}{2}\sum_1^2 (GMPE_i - GMPE_j)^2}$

$$\min E = \frac{1}{\sum_{i<j} \varepsilon_{ij}} \sum_{i<j} \frac{(\varepsilon_{ij} - \delta_{ij}^{map})^2}{\varepsilon_{ij}}$$

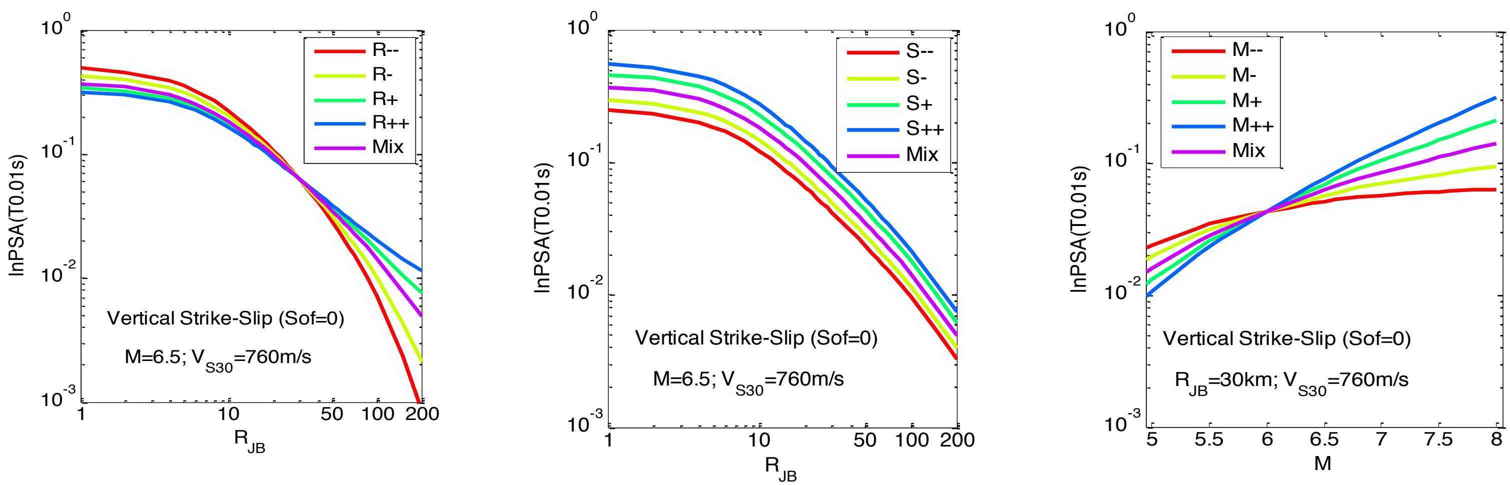
  - Visualize a set of GMPEs on the 2-D map
    - Similarity/ dissimilarity
    - Range of epistemic uncertainty
    - Etc..,





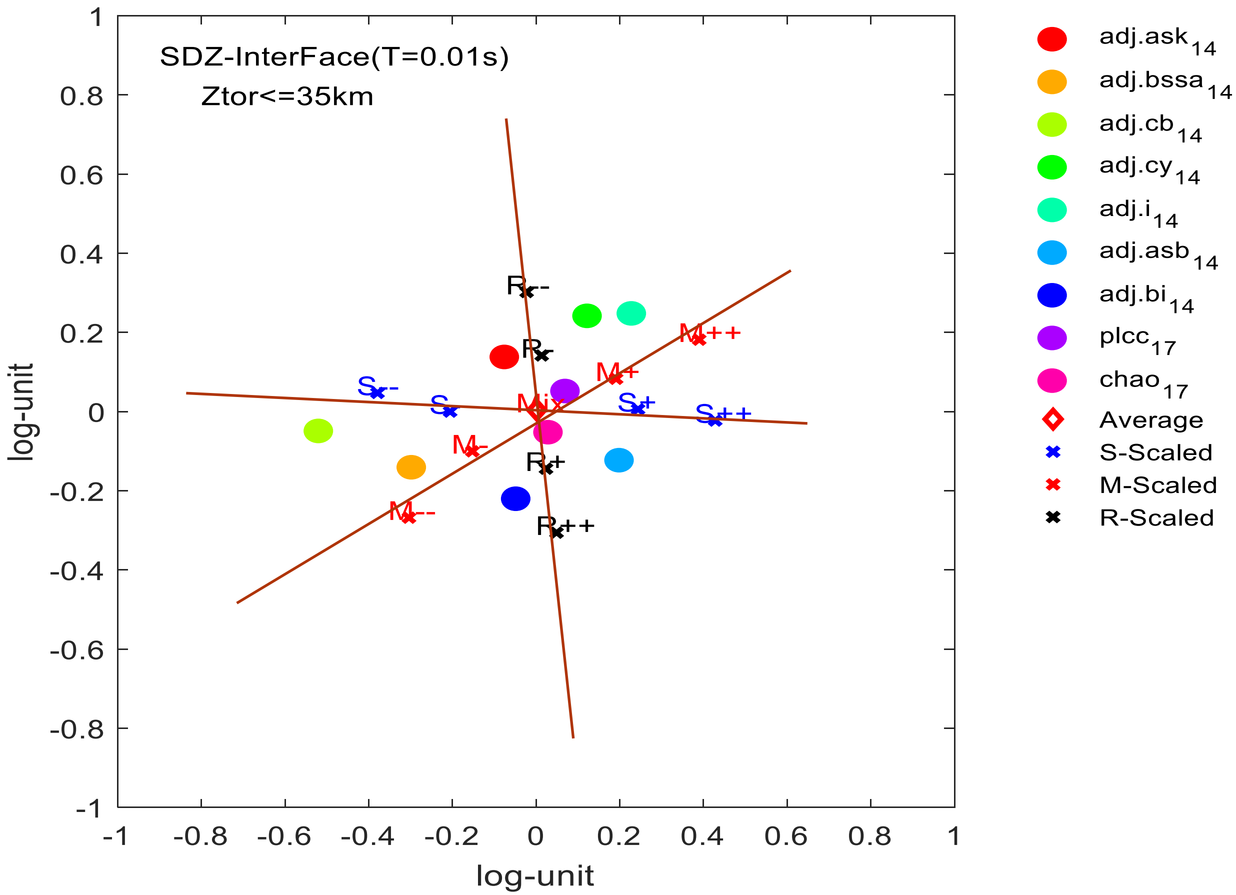
Visualization of GMPEs on Sammon Map

- To interpret the map, reference models are added:
  - The average of all candidate models:
    - Mix =1/N($\sum_{i=1}^{N} GMPE_i\,(M, R, \theta)$)
  - Up-Down scaled:
    - Mix+logα with α={0.67, 0.8, 1.25, 1.5}
  - Magnitude Scaled:
    - Mix+β(M-6) with β = {-0.4, -0.2 , 0.2, 0.4}
  - Distance Scaled:
    - Mix+γ(R-30) with γ = {-0.005, -0.0025, 0.0025, 0.005}



"Working title" Visualization of the range of epistemic uncertainty associated with GMPEs for PSHA". (N.M. Kuehn, et al.,(2015)

Visualization of GMPEs on Sammon Map



Visualize a set of GMPEs on the 2-D map

- Similarity/ dissimilarity
- Range of epistemic uncertainty

Etc..,

Ref: "Working title visualization of the range of epistemic uncertainty associated with GMPEs for PSHA". (N.M. Kuehn, et al.,(2015))

Common Form SCR

- Common Functional Form:

$$lnSA_{ref}(M, R_{RUP}, Z_{TOR}, V_{VS30} = 760, T)$$

$$= \theta_1(T) - \theta_8^2(T)R_{RUP} + \theta_9 Z_{TOR} + \left(\theta_5(T) + \theta_6(T)(M-5)\right)ln\left(\sqrt{R_{RUP}^2 + \theta_7^2(T)}\right)$$

$$+ \begin{cases} \theta_2(Mc_1 - Mc_2) + \theta_3(M - Mc_1) & for\ M < Mc_1 \\ \theta_2(M - Mc_2) & for Mc_1 \le M < Mc_2 \\ \theta_4(M - Mc_2) & for\ M \ge Mc_2 \end{cases}$$

A total of 11 model parameters in the common form

Constraints:

Positive magnitude scaling ratio: $\frac{\partial \ln(SA_{ref})}{\partial M} > 0$

Negative distance scaling ratio: $\frac{\partial \ln(SA_{ref})}{\partial R_{rup}} < 0$

Distance saturation of ground motion: $\theta_6 > 0$

Sampling of vector of GM values for fitting

- Vertical Strike slip (λ=0, δ=90^0), V_{S30} = 760m/s:
  - M = 5.0, 5.2, 5.4, 5.5, 5.6, 5.8, 6.0, 6.2, 6.4, 6.5, 6.6, 6.8, 7.0, 7.2, 7.4, 7.5, 7.6, 7.8, 8.0.
  - R_{JB} =1, 2, 4, 5, 6, 8, 10, 12, 14, 15, 16, 18, 20, 22, 24, 25, 26, 28, 30, 35, 40, 45, 50, 55, 60, 65, 70, 80, 90, 100, 150, 200.

  - Z_{TOR}-M_W Relationship (CY14):

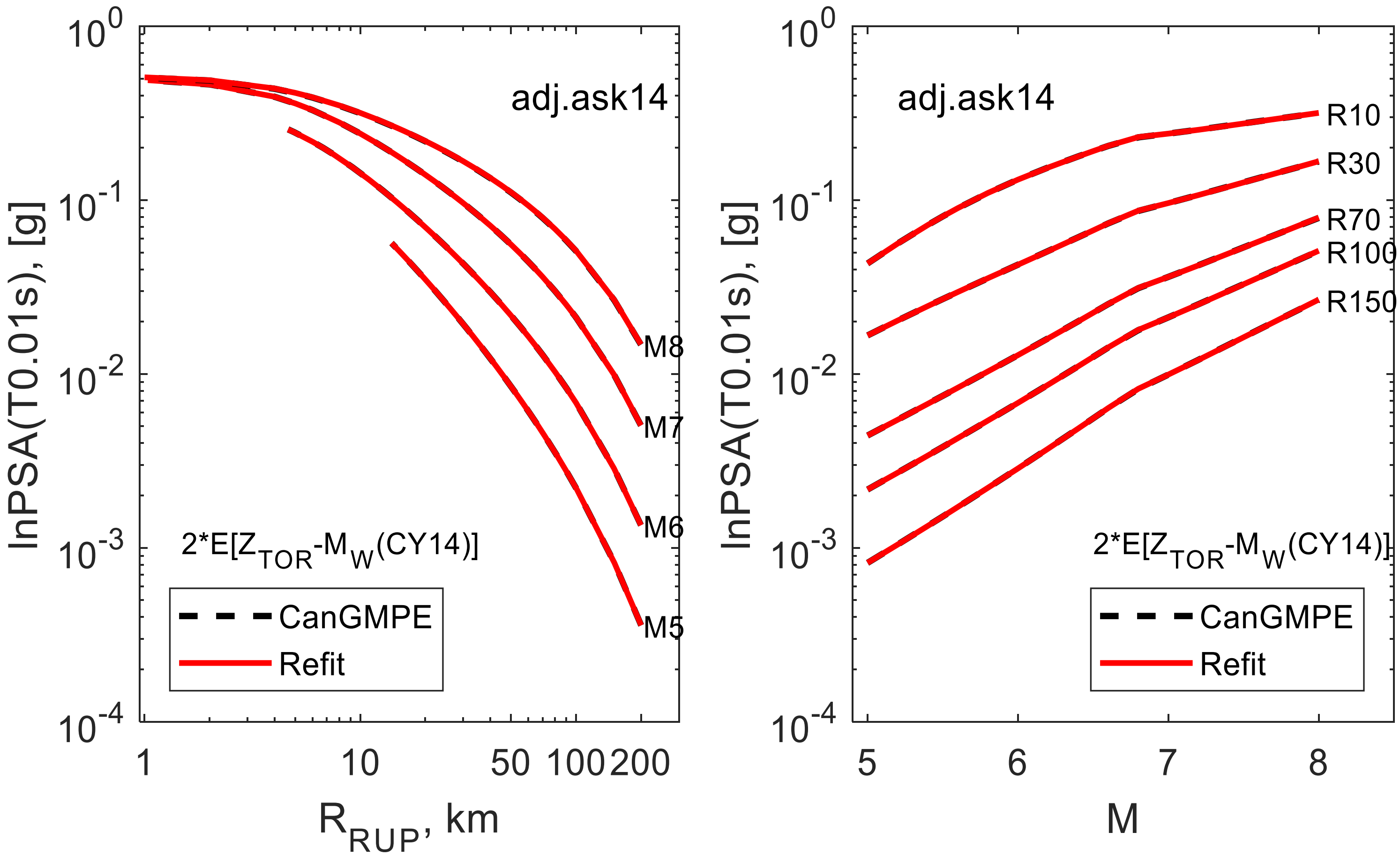
    For strike slip and normal:

    $$EZ_{TOR} = \text{mul} * (\max(2.673 - 1.136\max(M - 4.970,0), 0))^2$$

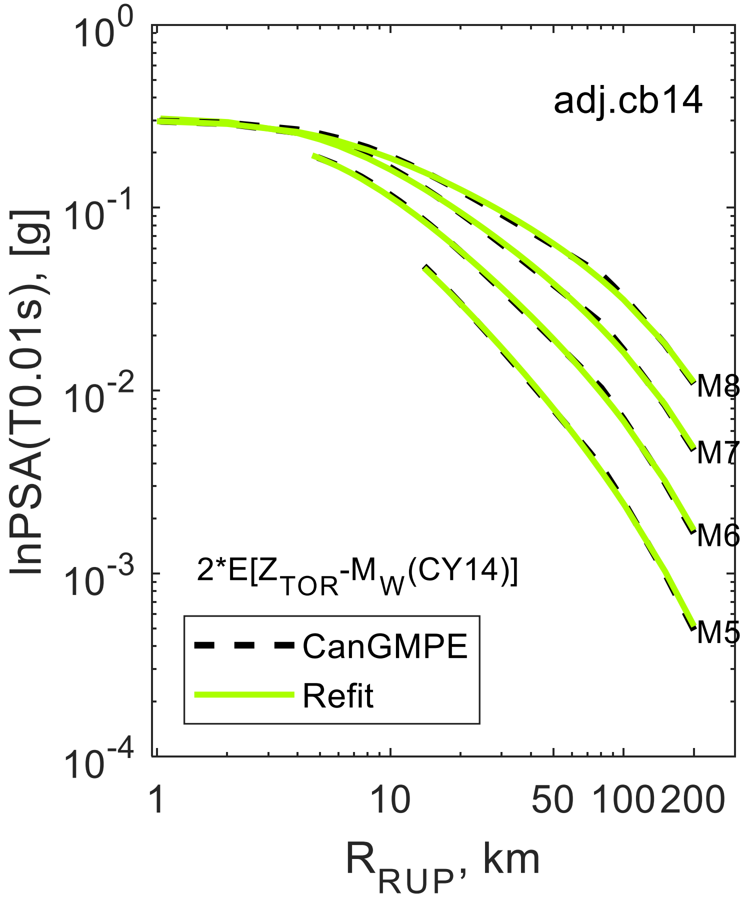
    - Consider uncertainty (multiplier = 0.5, 1, 2, 3, 5)
  - $$R_{RUP} = \sqrt{R_{JB}^2 + Z_{TOR}^2}$$

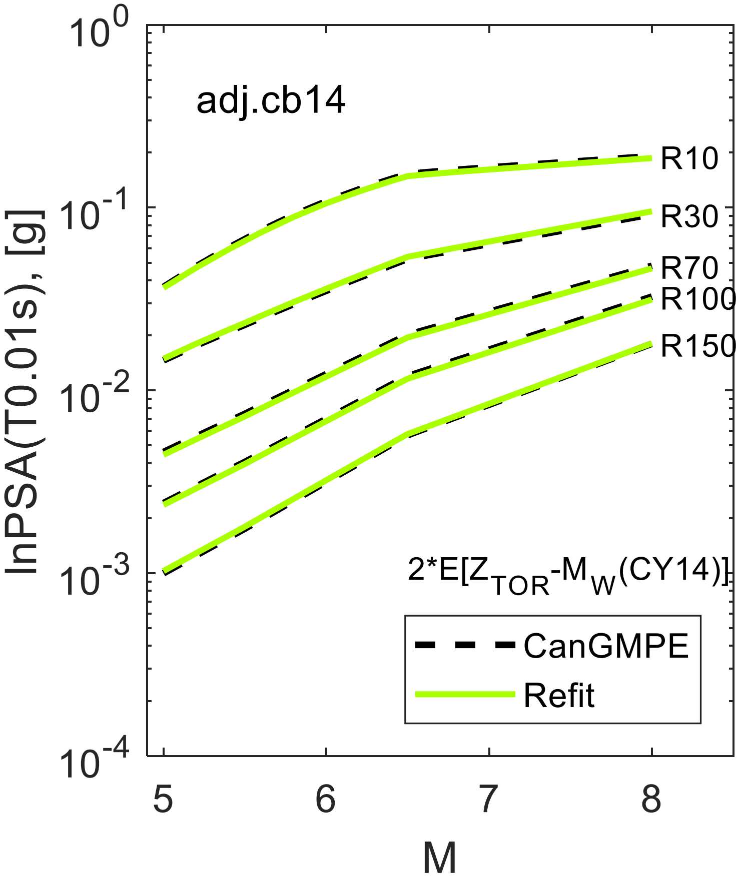
→ 1SOF *5Z_{TOR}*19M*32R=3040 scenarios

Comparison between the original and the refit candidate GMPE (ASK 14)

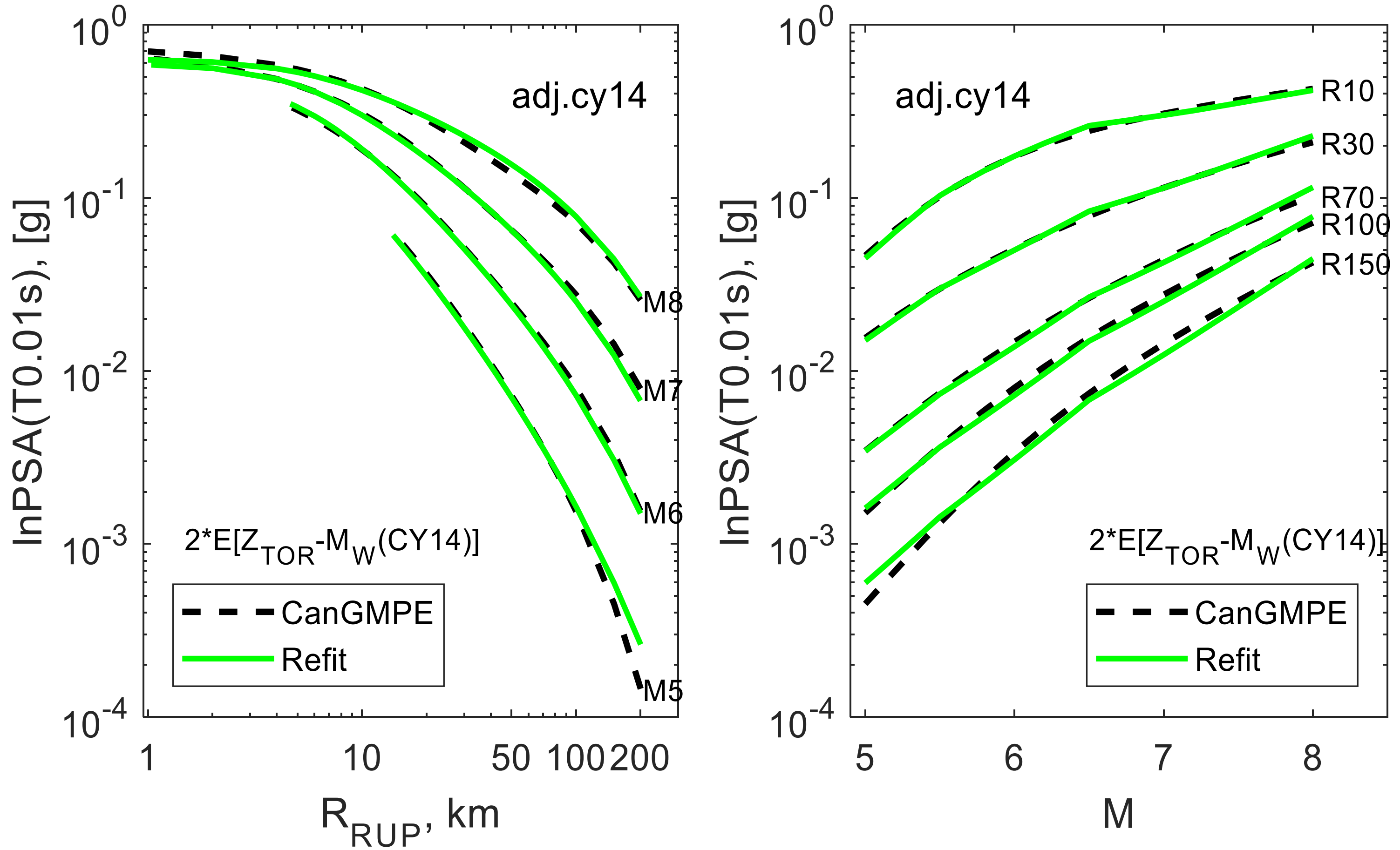


Comparison between the original and the refit candidate GMPE (CB 14)

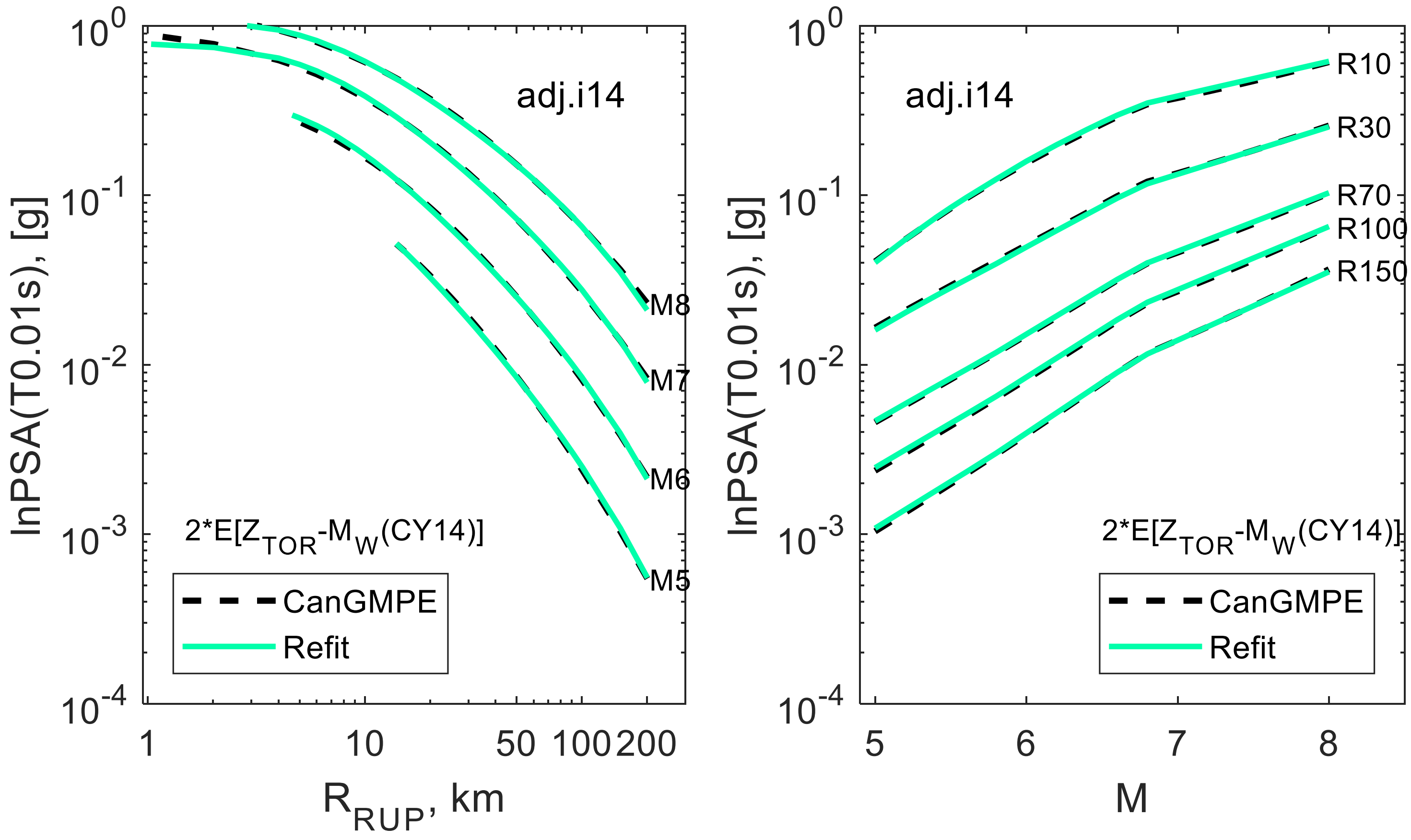




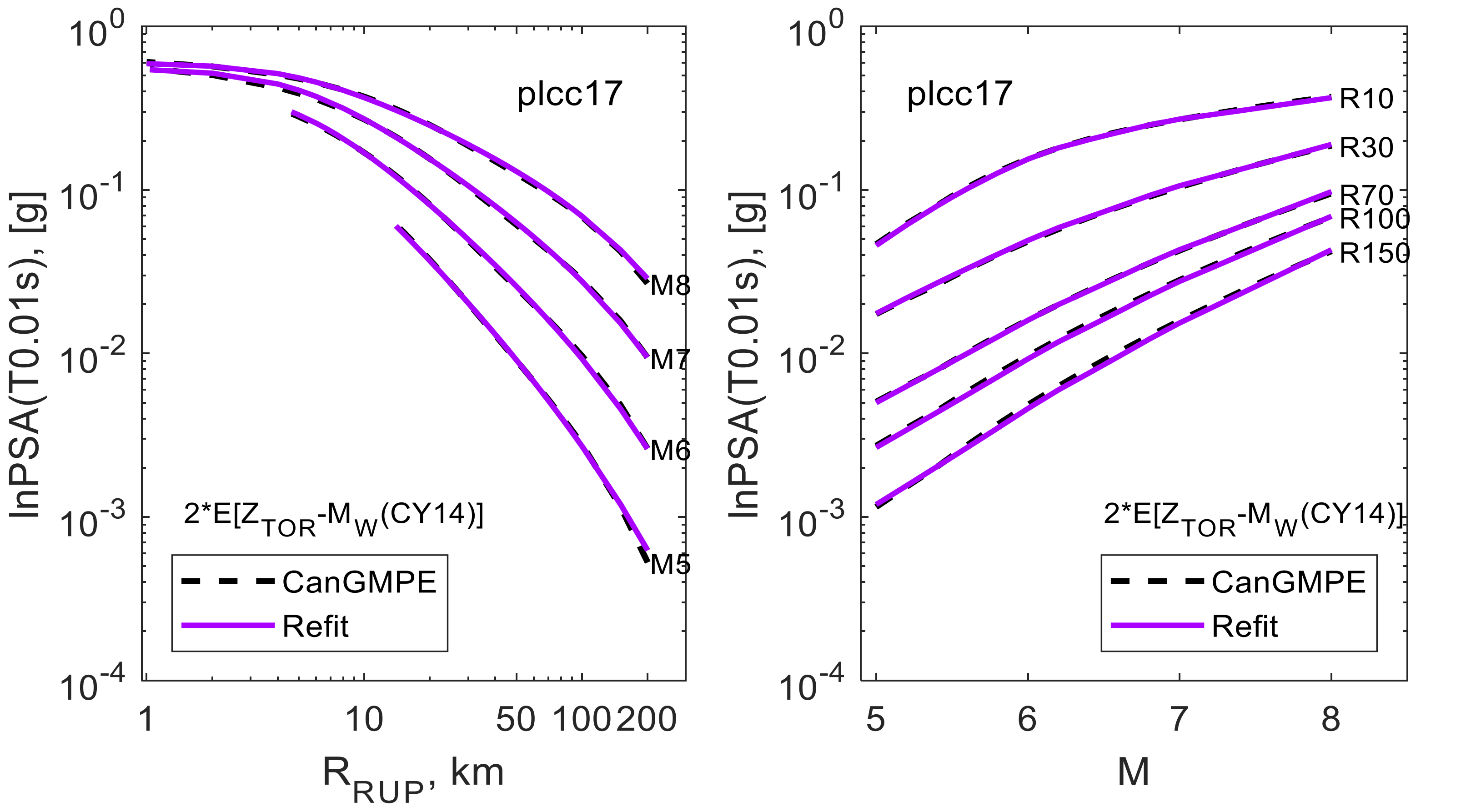
Comparison between the original and the refit candidate GMPE (CY 14)



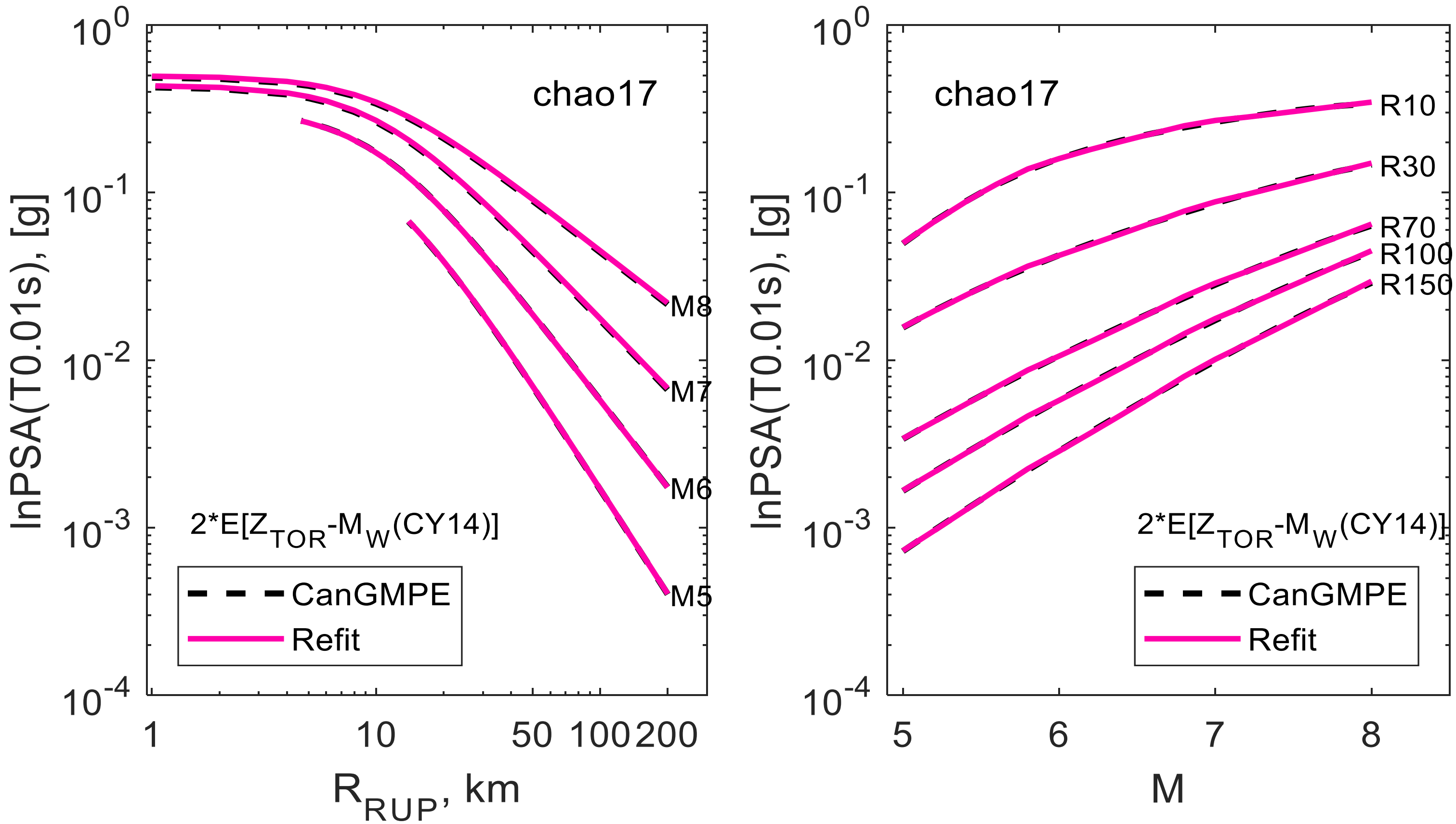
Comparison between the original and the refit candidate GMPE (I14)



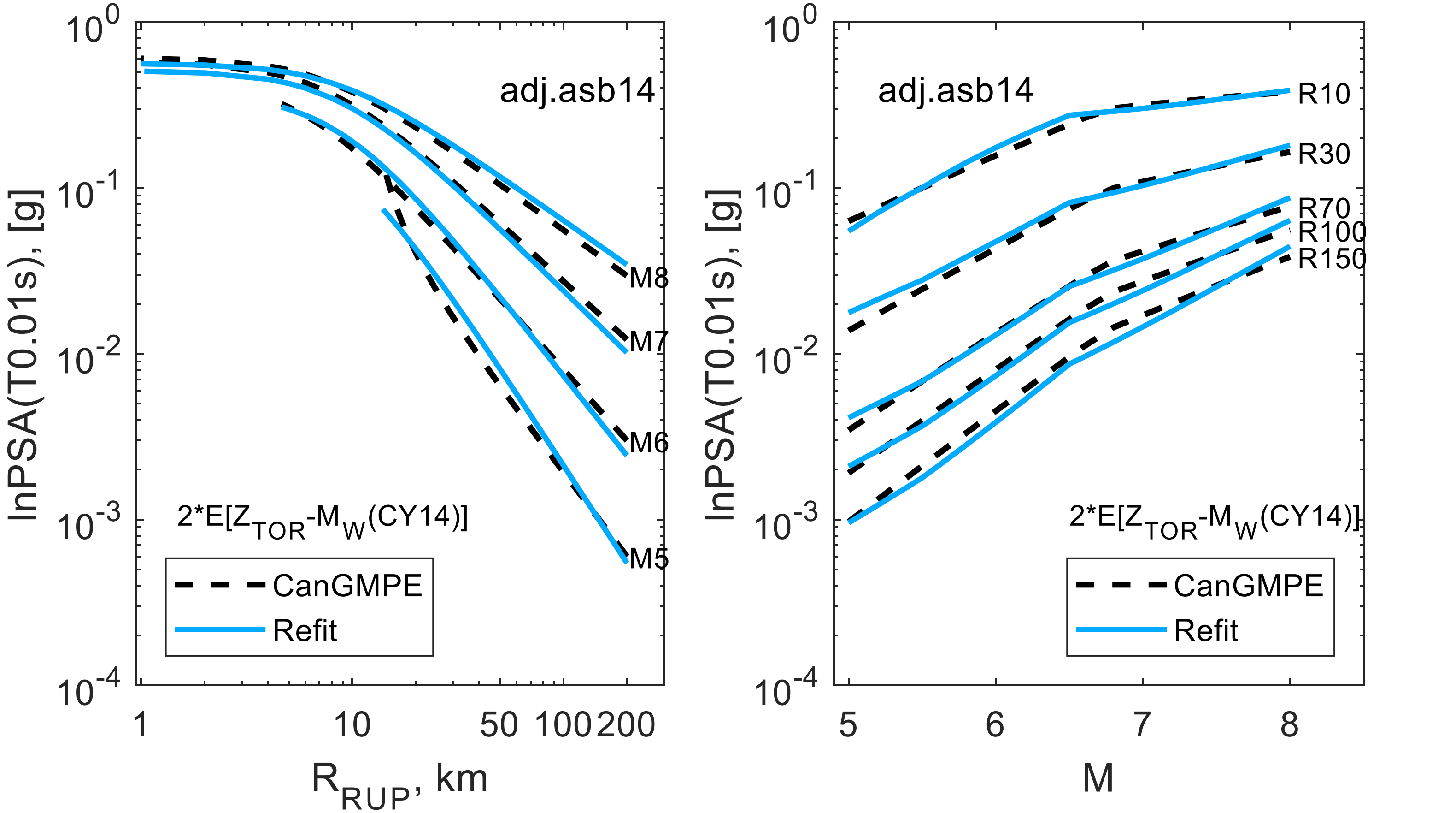
Comparison between the original and the refit candidate GMPE (PLCC 17)



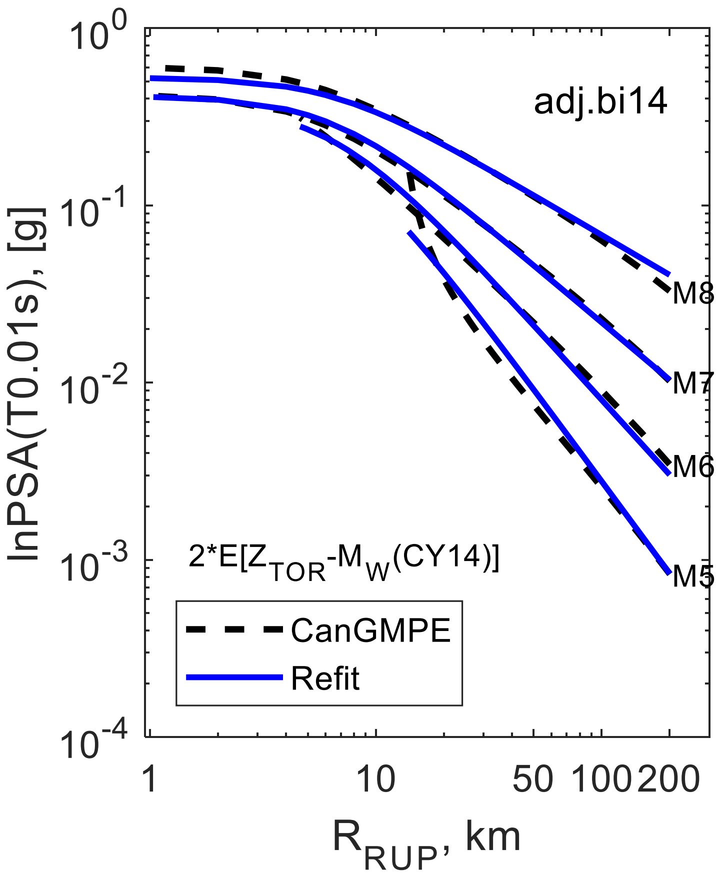
Comparison between the original and the refit candidate GMPE (CHAO 14)

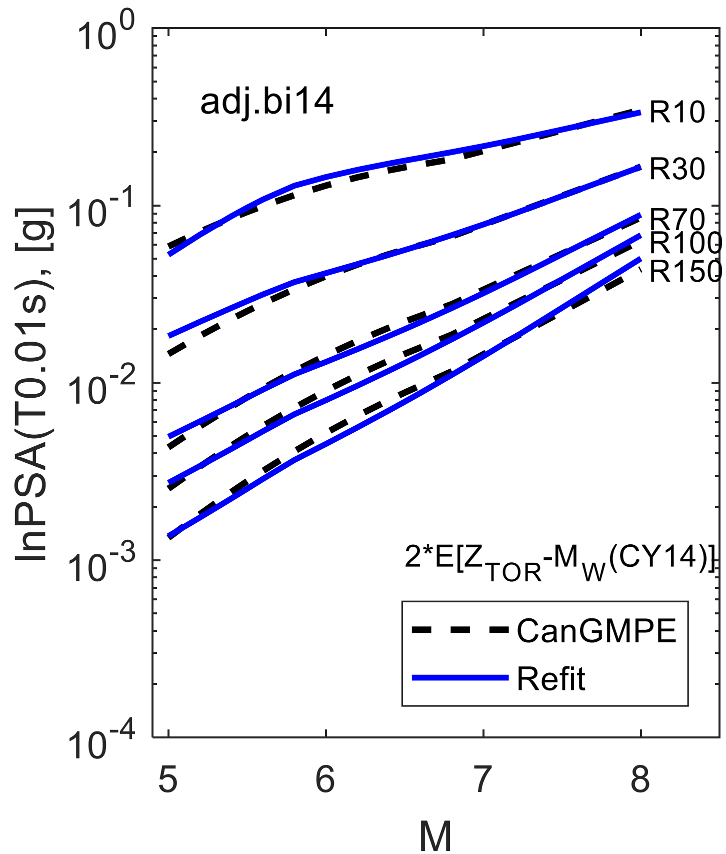


Comparison between the original and the refit candidate GMPE (ASB 14)

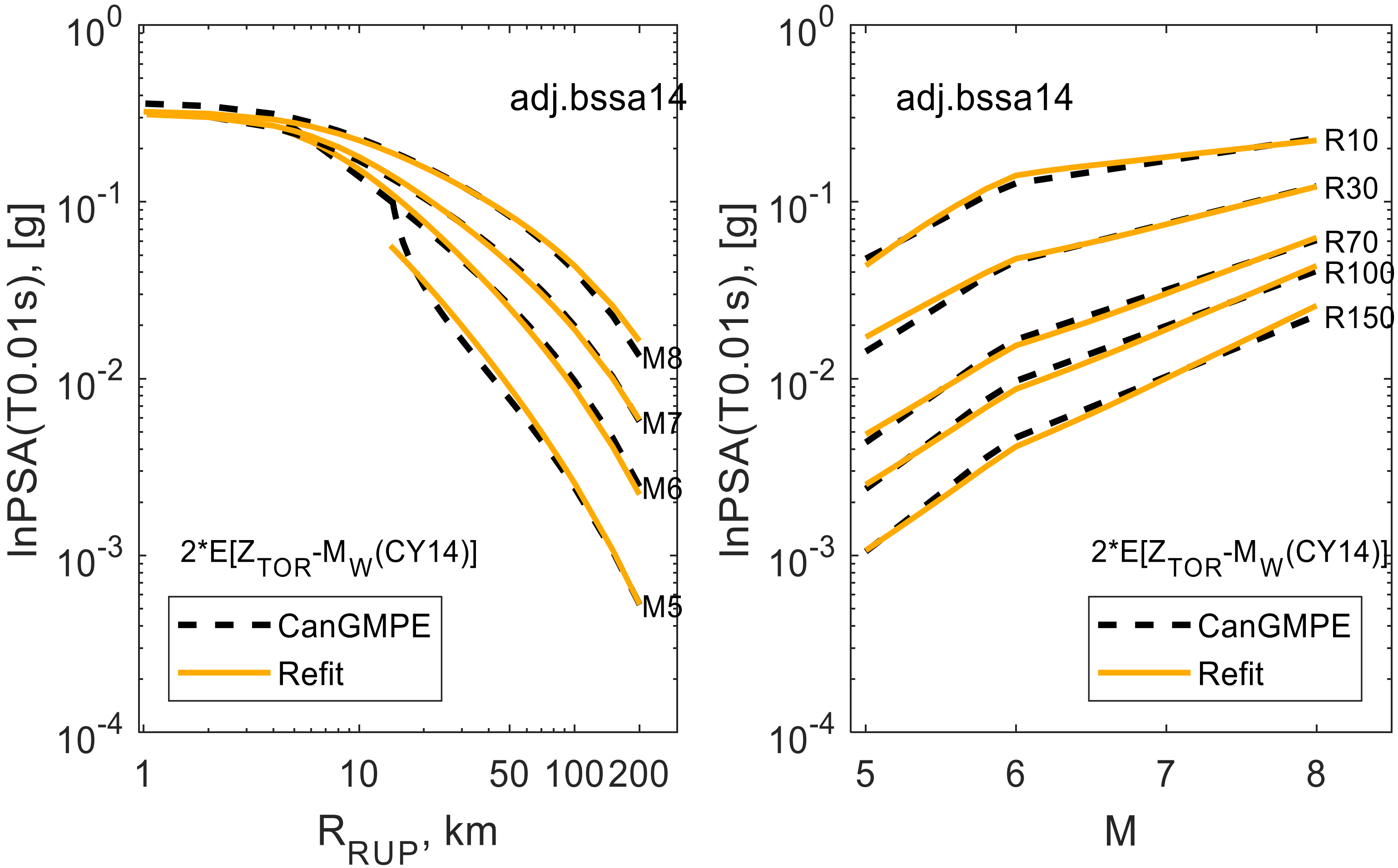


Comparison between the original and the refit candidate GMPE (BI 14)





Comparison between the original and the refit candidate GMPE (BSSA 14)



Continuous Distributions of the Median Prediction

In order to increase the correlation among coefficients, we can obtain more set of coefficients by fitting the common form to the interpolated GM

$$Interp(lnSA(M,R)) = w_1 ln(SA_i) + w_2 ln(SA_j)$$

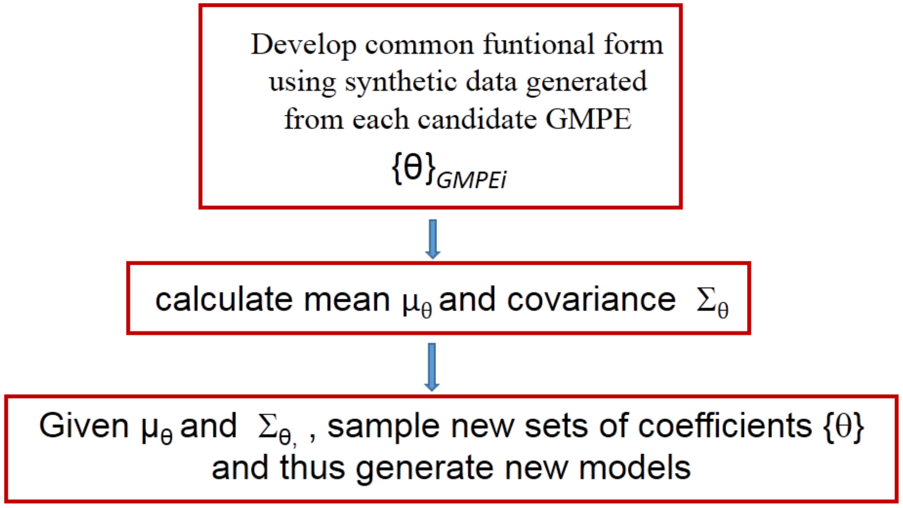
$$w = \left\{\frac{1}{3}, \frac{2}{3}\right\}, \left\{\frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{2}{3}, \frac{1}{3}\right\}$$

$$\{\theta_{GMPEi}\}_{(9+108)\times 11} \rightarrow \begin{Bmatrix} \mu_\theta \\ \Sigma_\theta \end{Bmatrix}$$

$9 + {}_9C_2 = 9 + 108$

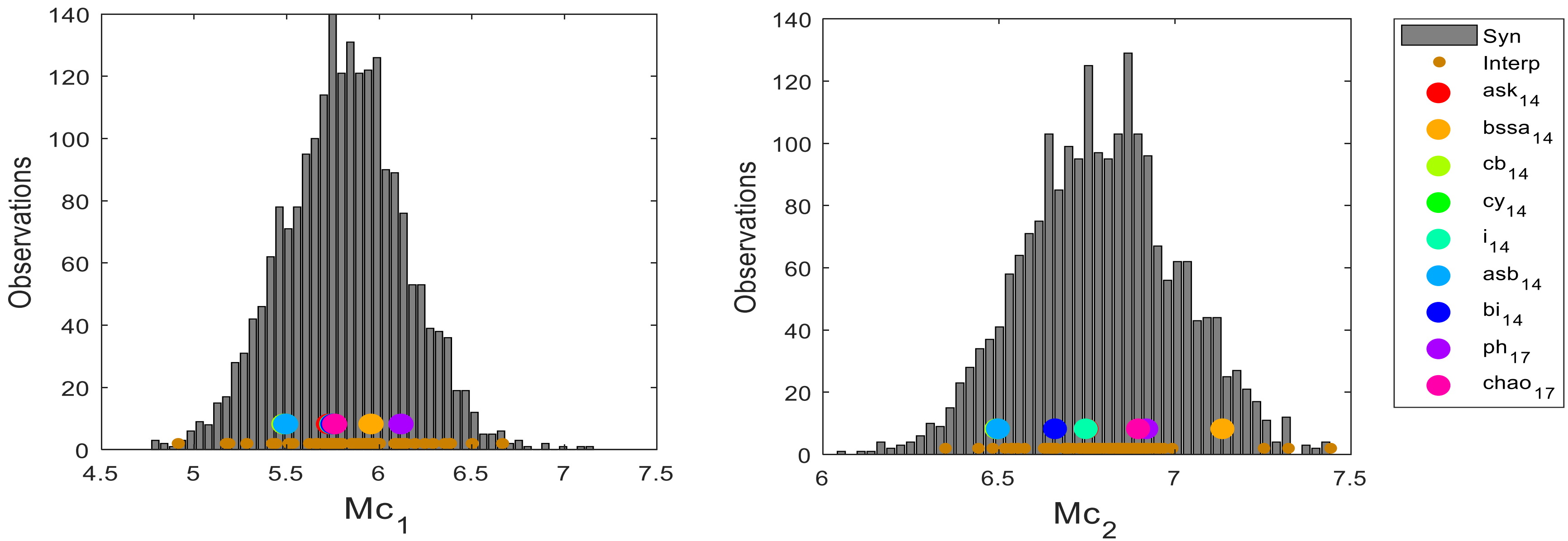
- Estimate and Sample of

  the coefficient Covariance Matrix



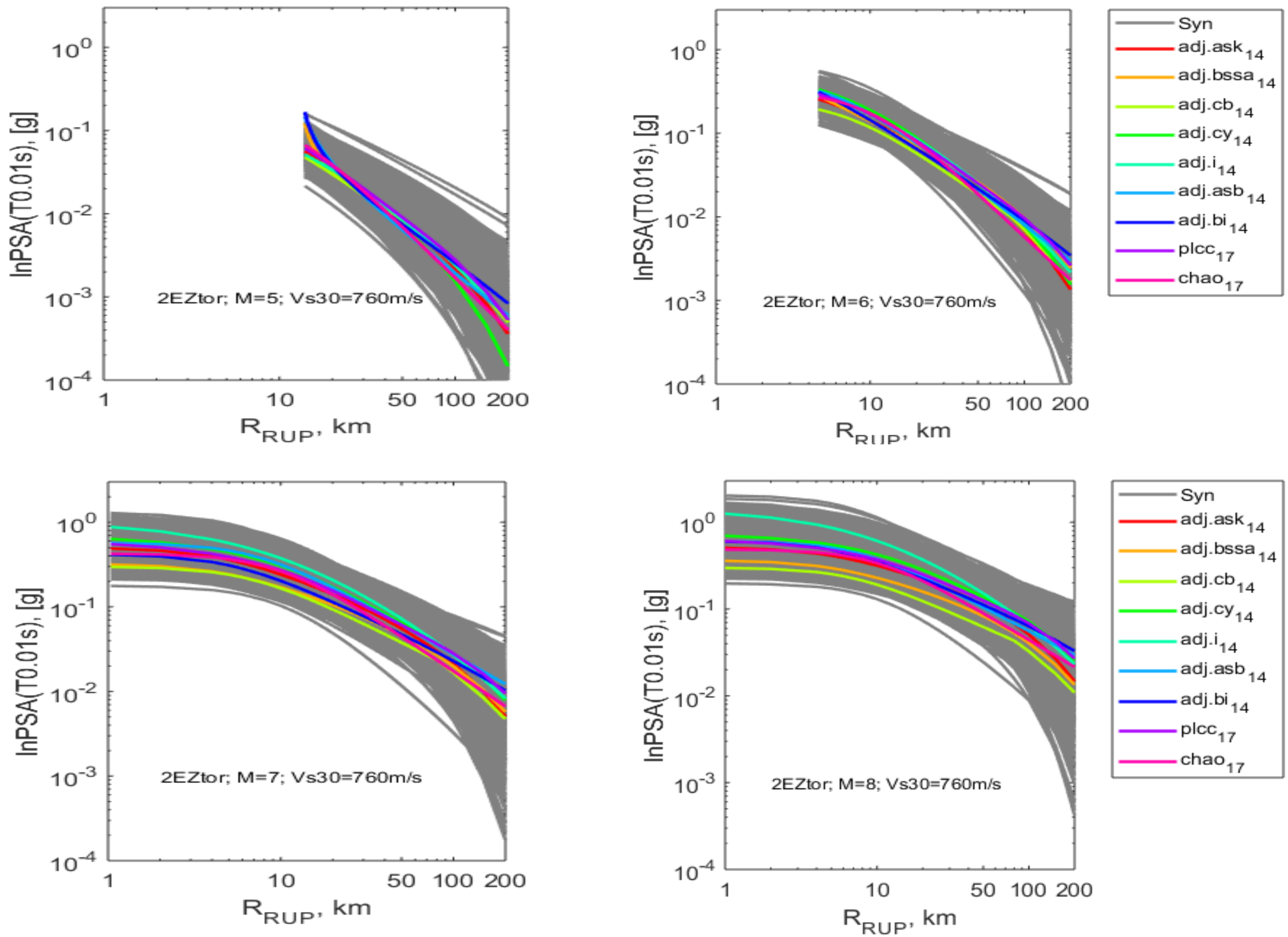
Refit to Interpolated GM

- Sample the number of synthetic GMPE models from $\begin{Bmatrix} \mu_\theta \\ \Sigma_\theta \end{Bmatrix}$
  - Number of models =2000
  - Range of models is broaden using $2\Sigma_\theta$



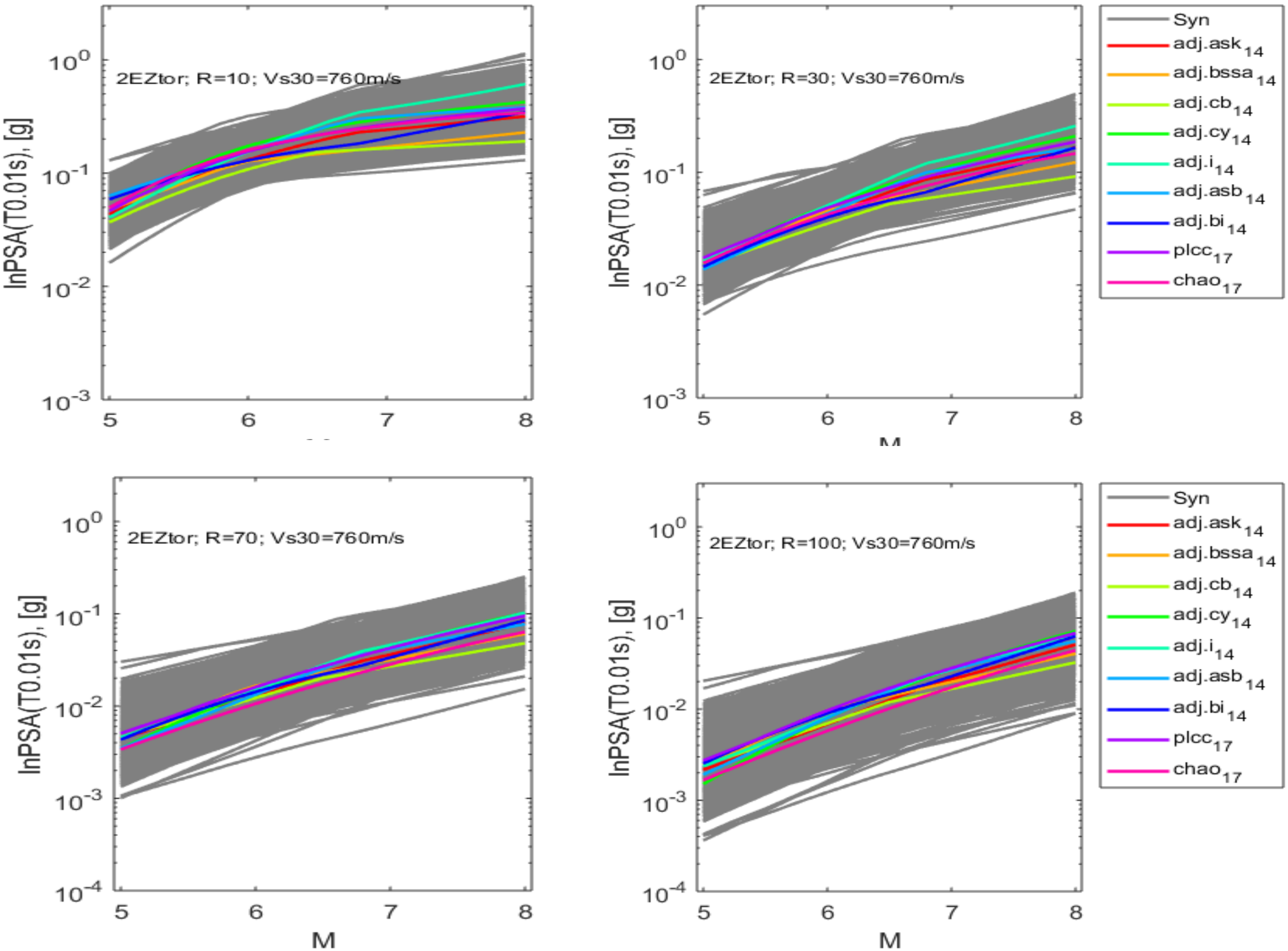
Range of GMPE

$$\begin{Bmatrix} \mu_\theta \\ 2\Sigma_\theta \end{Bmatrix}$$



Range of GMPE

$$\begin{Bmatrix} \mu_\theta \\ 2\Sigma_\theta \end{Bmatrix}$$



Range of GMPE on 2-D Hazard Space

- Hazard Space~ Consistent with de-aggregation bin:
  - Vertical Strike slip (λ=0, δ=90^0), V_{S30}= 760m/s:
    - M = 5.1, 5.3, 5.5, 5.7,5.9, 6.1, 6.3, 6.5, 6.7, 6.9, 7.1, 7.3, 7.5, 7.8, 8.3
    - R_{RUP}=1, 3, 5, 7, 9, 12, 16, 20, 24, 28, 32.5, 37.5, 42.5, 47.5, 60, 80, 95, 125, and 236.6 km
  - $Z_{TOR} \leq 35km \sim 100\%$ contribution
  - $R_{JB} = |R_X|$ *and* $R_{JB} = \sqrt{R_{Rup}^2 - Z_{TOR}^2}$
  - $R_{RUP} \geq Z_{TOR}$
  - Four NPP(1~4) sites
    - Average Hazard contribution

→ Number of Scenarios= 2464

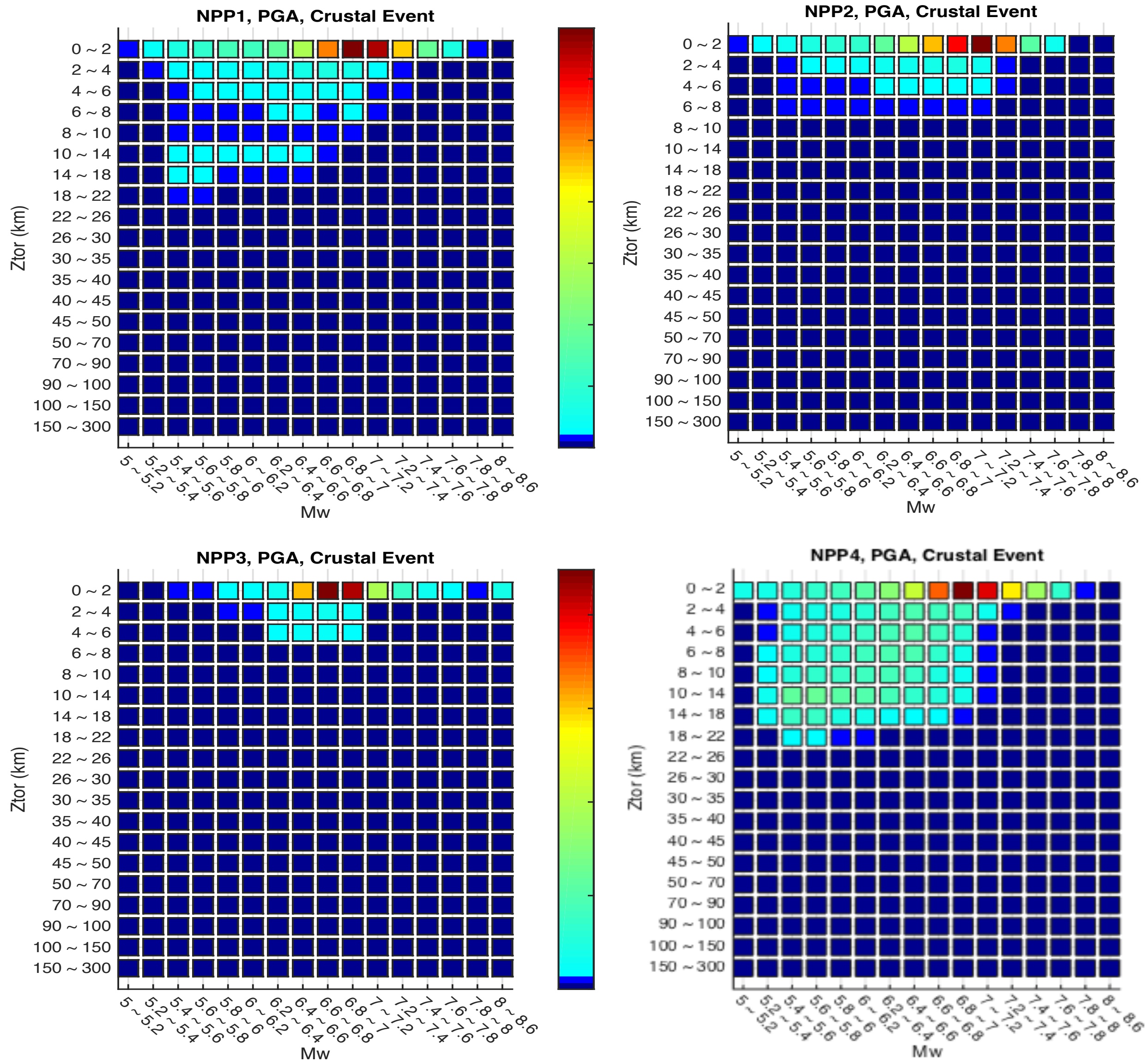
Range of GMPE on 2-D Hazard Space

Consider:

Average Hazard contribution

$$Z_{TOR} \leq 35km$$

$\sim 100\%$ contribution



Mapping GMPEs on Sammon Map

Sammon's map configuration:

$$\min E = \frac{1}{\sum_{i<j} \varepsilon_{ij}} \sum_{i<j} \frac{(\varepsilon_{ij} - \delta_{ij}^{map})^2}{\varepsilon_{ij}}$$

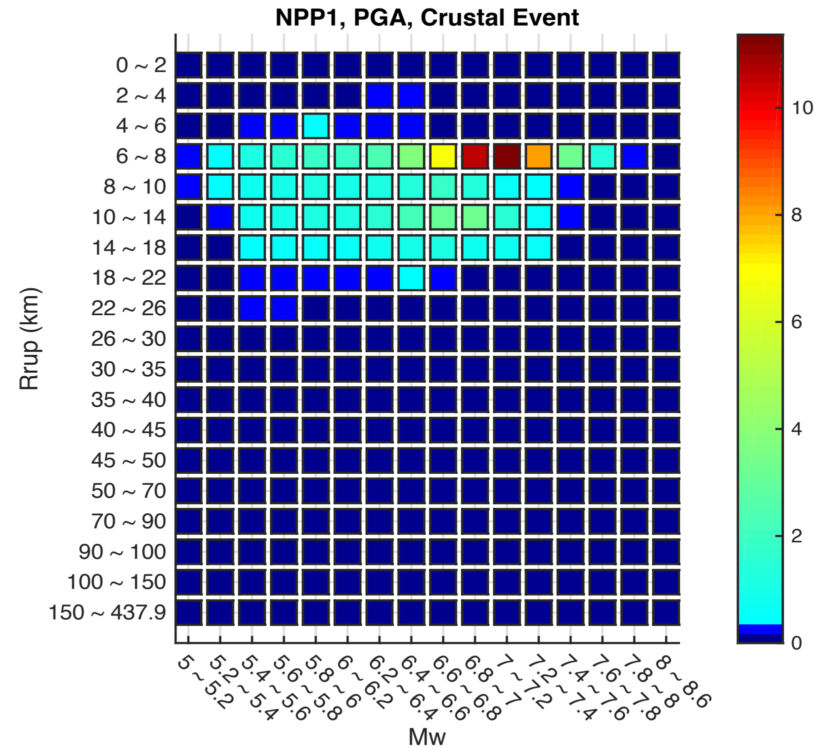
Where Euclidian distance (ε_{ij}) is weighted by hazard contribution w_i

$$\varepsilon_{ij} = \sqrt{\sum_{i=1}^{N} w_i (x_{i,1} - x_{i,2})^2}$$

The renormalized weights:

$$W_i = 0.5\left(W_{DEAG_{ik}} + \frac{1}{NS}\right)$$

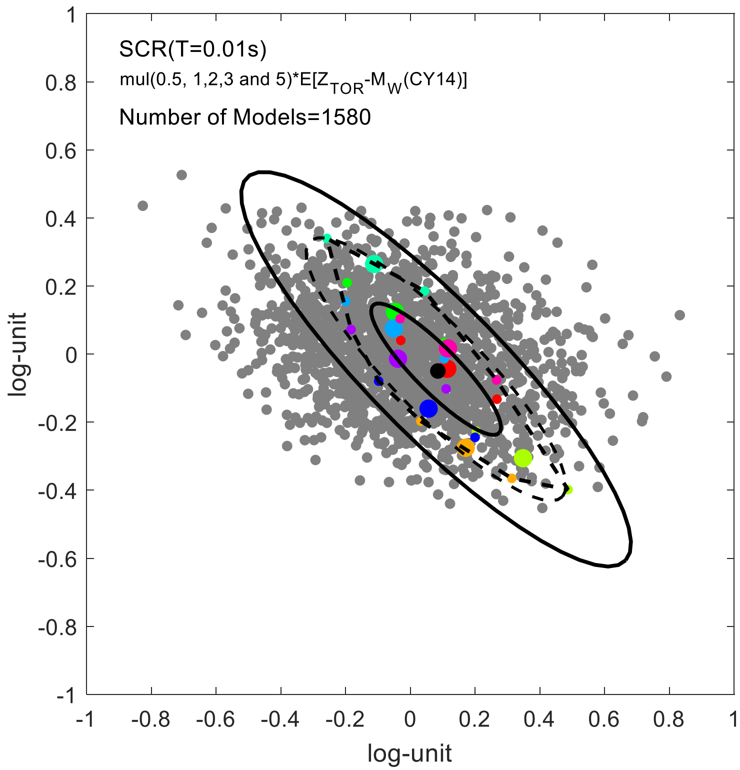
where NS is the total number of scenarios.

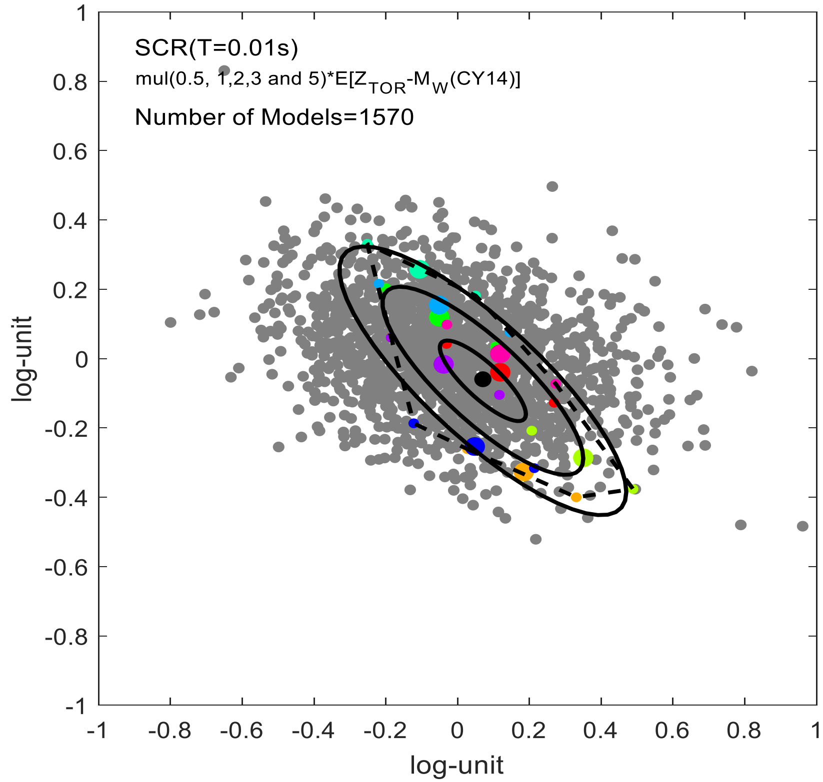


Range of GMPE

Project on the 2D Sammon Map

- Nine Candidate GMPEs
- Nine Candidate GMPEs $\pm 2\sigma_{AY14}$
- Two thousand synthetic models





Fitted ellipse based on candidate models.
Inner ellipse~0.5 scaled down from the fitted ellipse
Outer ellipse~1.5 scaled up from the fitted ellipse
(~ SWUS report)

Fitted ellipse based on candidate models.
Inner-1 ellipse~0.3 scaled down from the fitted ellipse
inner-2 ellipse~0.7 scaled up from the fitted ellipse

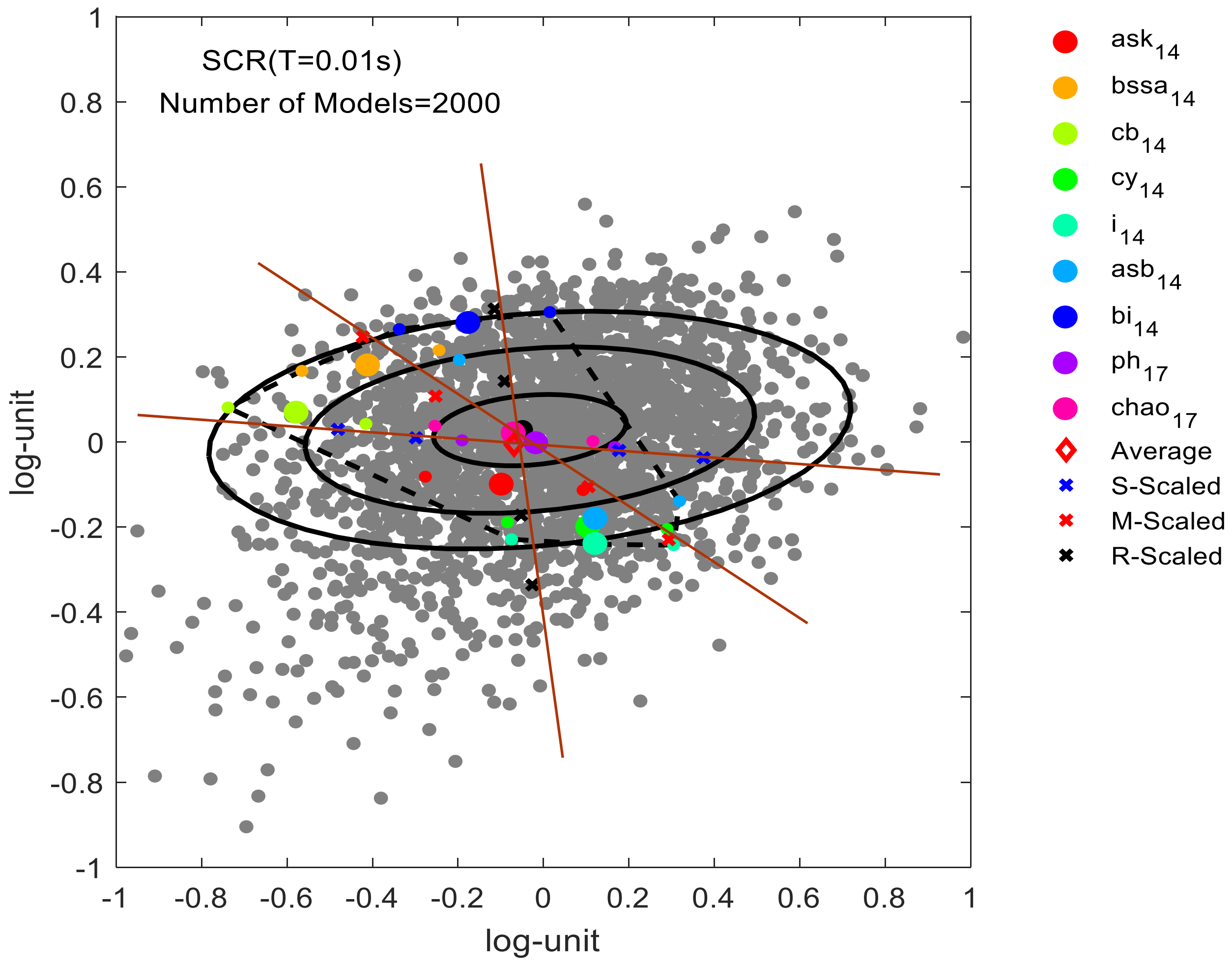
Range of GMPE

Project on the 2D Sammon Map

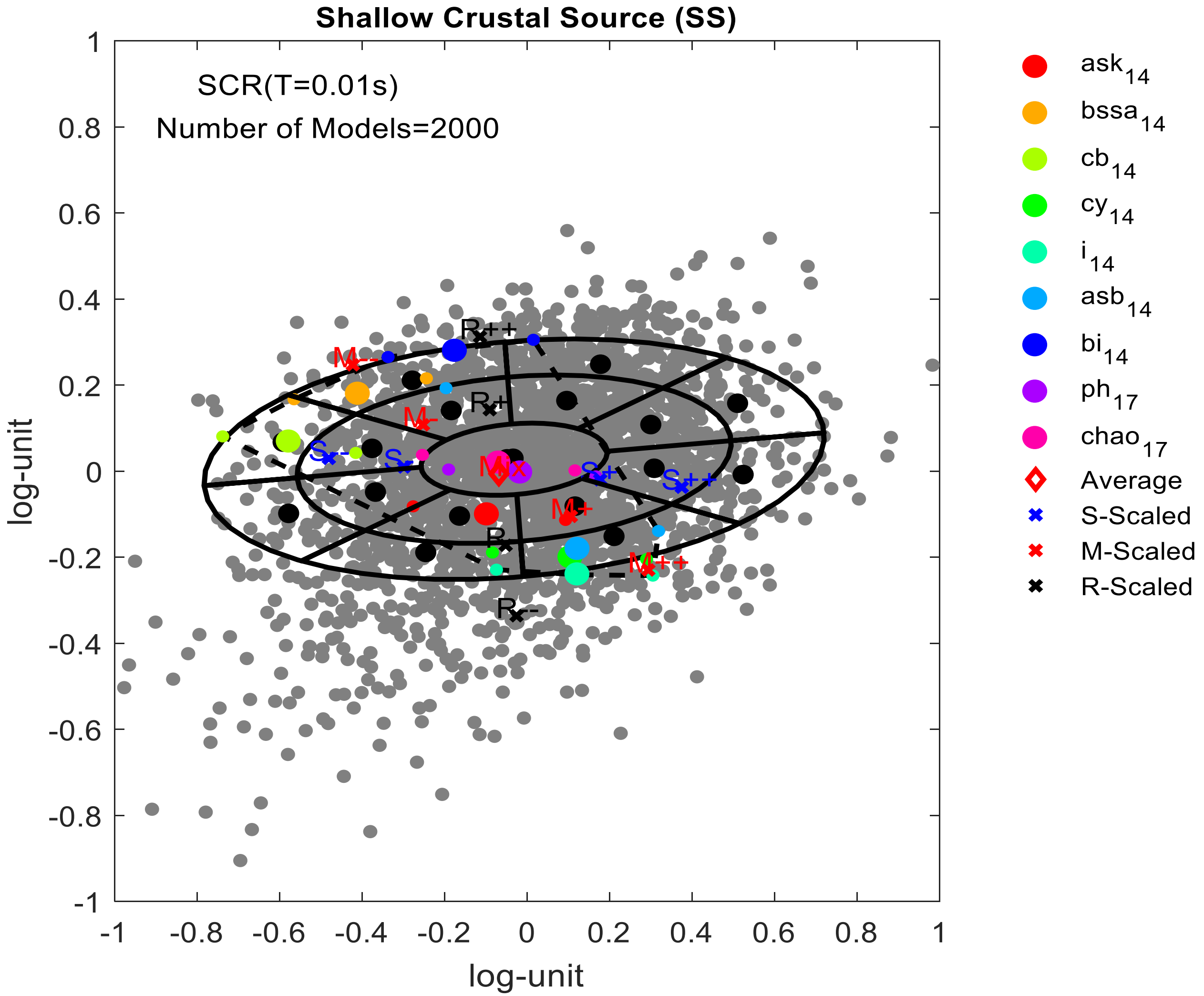
- Nine Candidate GMPEs
- Nine Candidate GMPEs $\pm 2\sigma_{AY14}$
- Two thousand synthetic models

Rotate the Map:

- Locate the mean of all models at the center {0,0}
- S-Scaling direction orient roughly along the x-axis

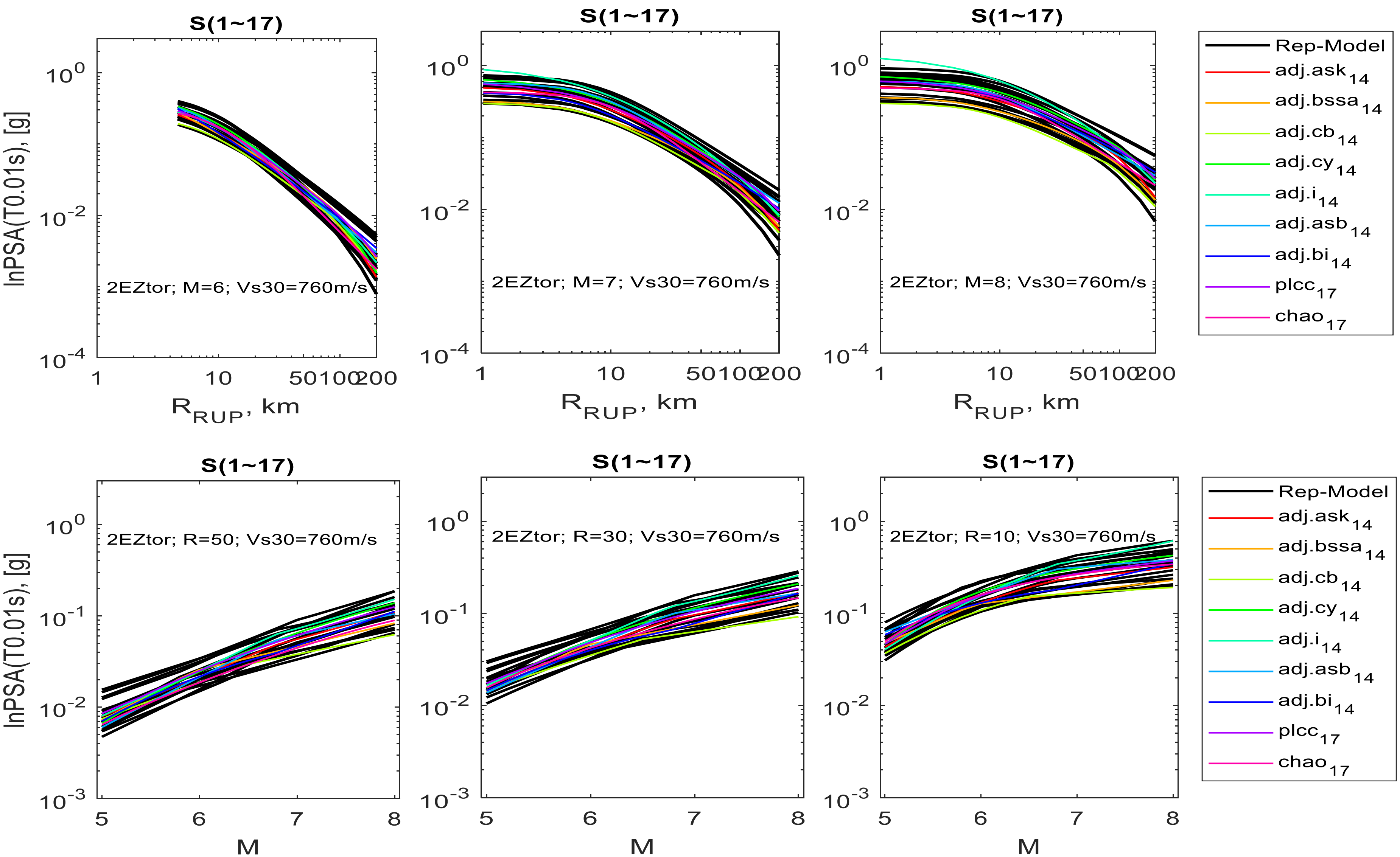


Representative Models

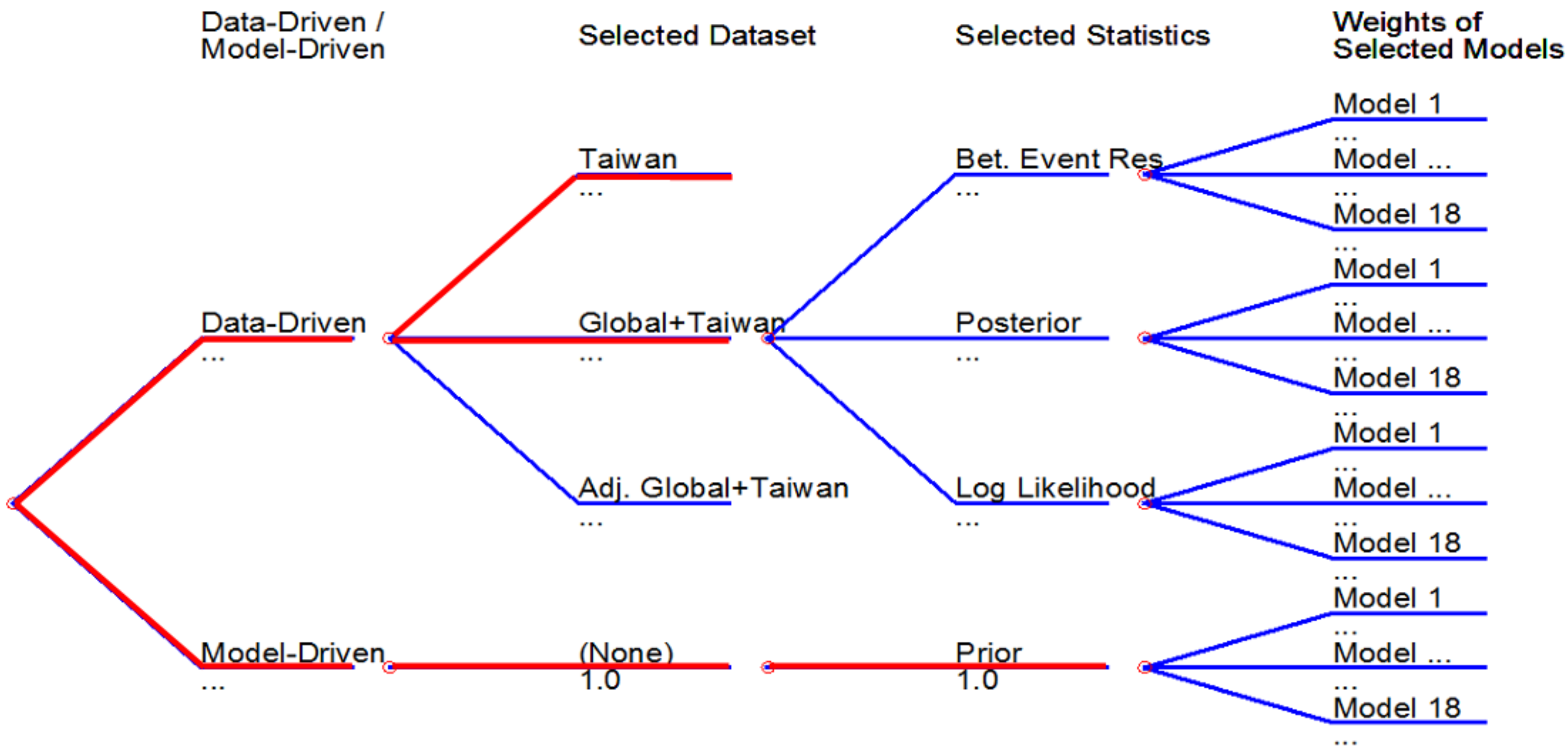


The models closest to the centroid are selected as the representative models

Representative Models



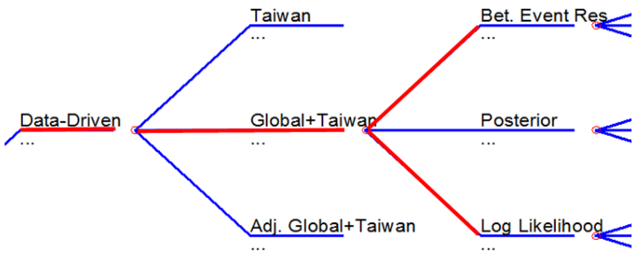
Weighting Scheme



Calculate the Weights

- Data Selection criteria.
- Examine models with respect to the selected data.
- Correct data to reference site (strike slip, $V_{S30} = 760 m/s$)
- Examine models with respect to the corrected data.
- Calculate the mean between event residuals and log-likelihood.
- Calculate weights
  - Residuals weight (wR)
  - Log-likelihood weight (wLL)
  - Prior weight (wPri)
  - Posterior weight (wPos)

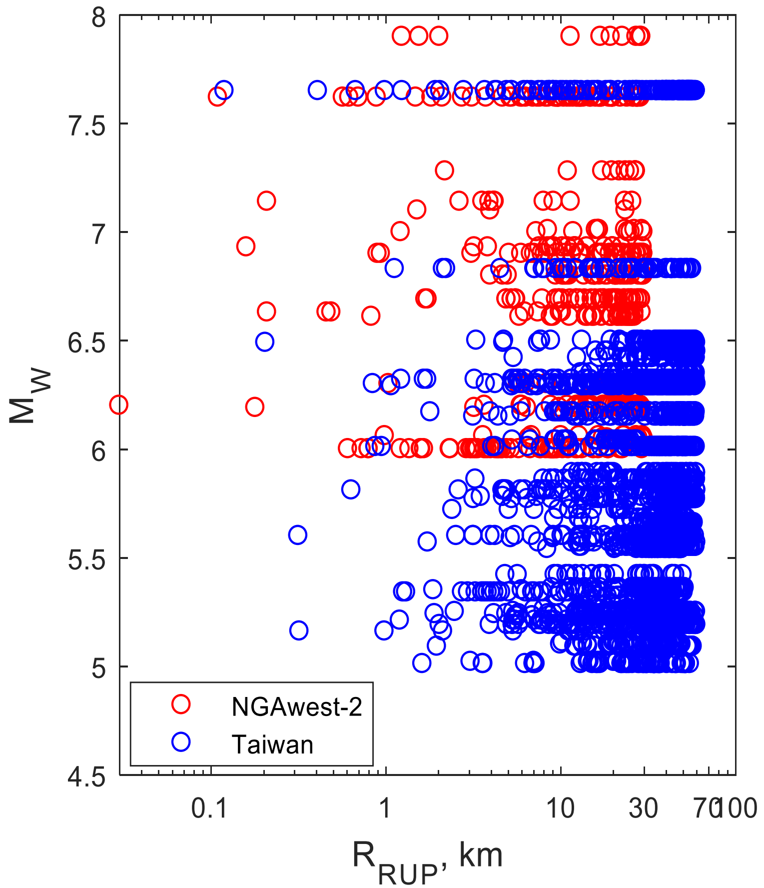
Data Selection Criteria



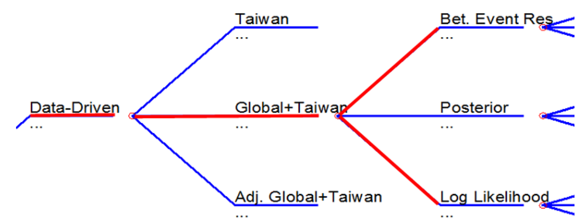
NGA-west2 and Taiwan.

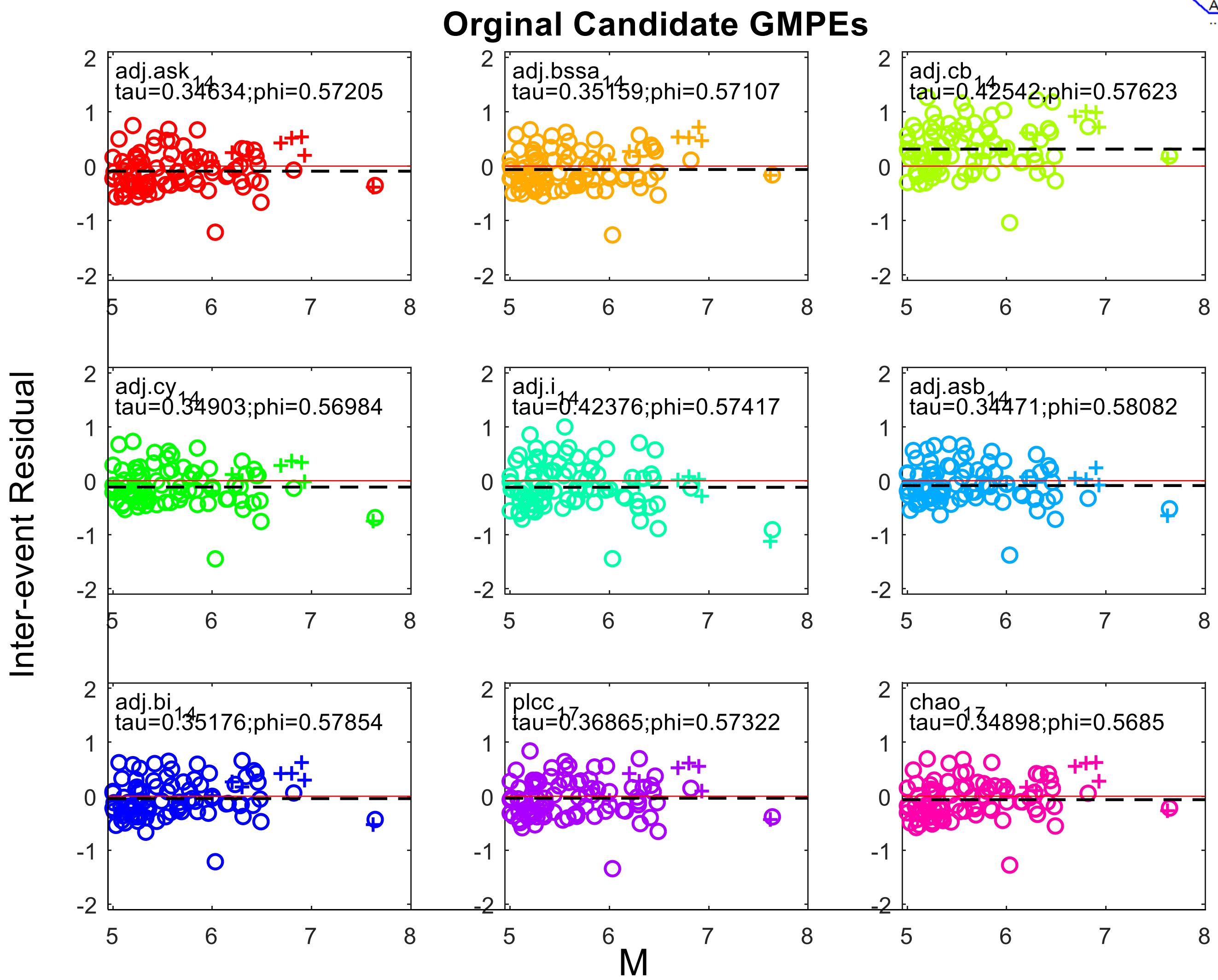
- Strike Slip, Reverse, and Normal.
- NGAwest-2
  - Mw≥ 6.0
  - $R_{RUP} \leq 30$km
- Taiwan
  - Mw≥ 5.0
  - $R_{RUP} \leq 60$km
- $V_{S30} \geq$300m/s
- At least 5 records/events
- At least 1 records within 20 km.

→151 events with 3121 records



Examine Candidate Models with respect to **truly recorded data**



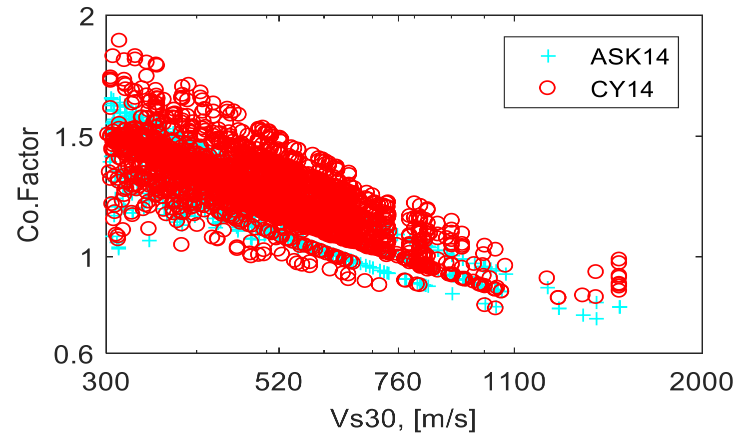


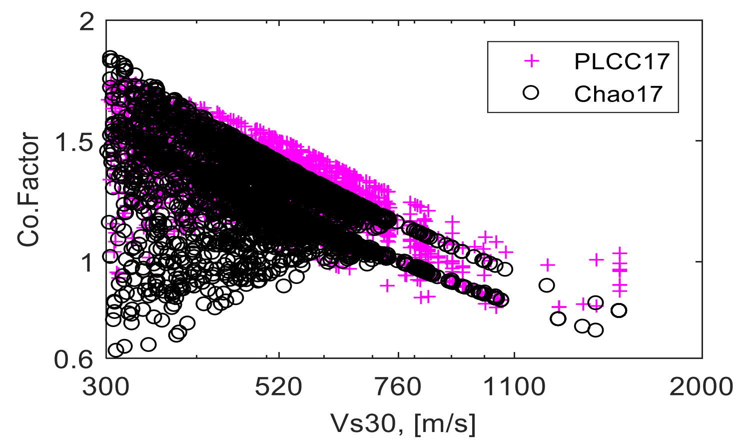
Data Correction to Reference Site

- Four models include nonlinear site effects
  - Adj-ASK14
  - Adj-CY14
  - PLCC17
  - Chao17
- Fault type corrected to reference fault type (SS)
- and Site corrected to reference VS30=760m/s
- Using four above-mentioned models:

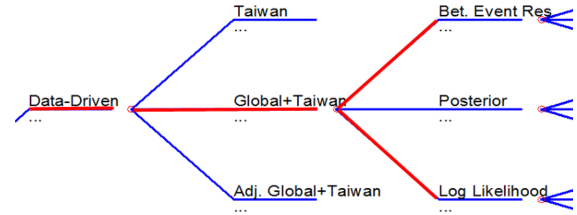
$$y_{ref.760} = \frac{y_{obs}}{\text{Correction Factor}}$$

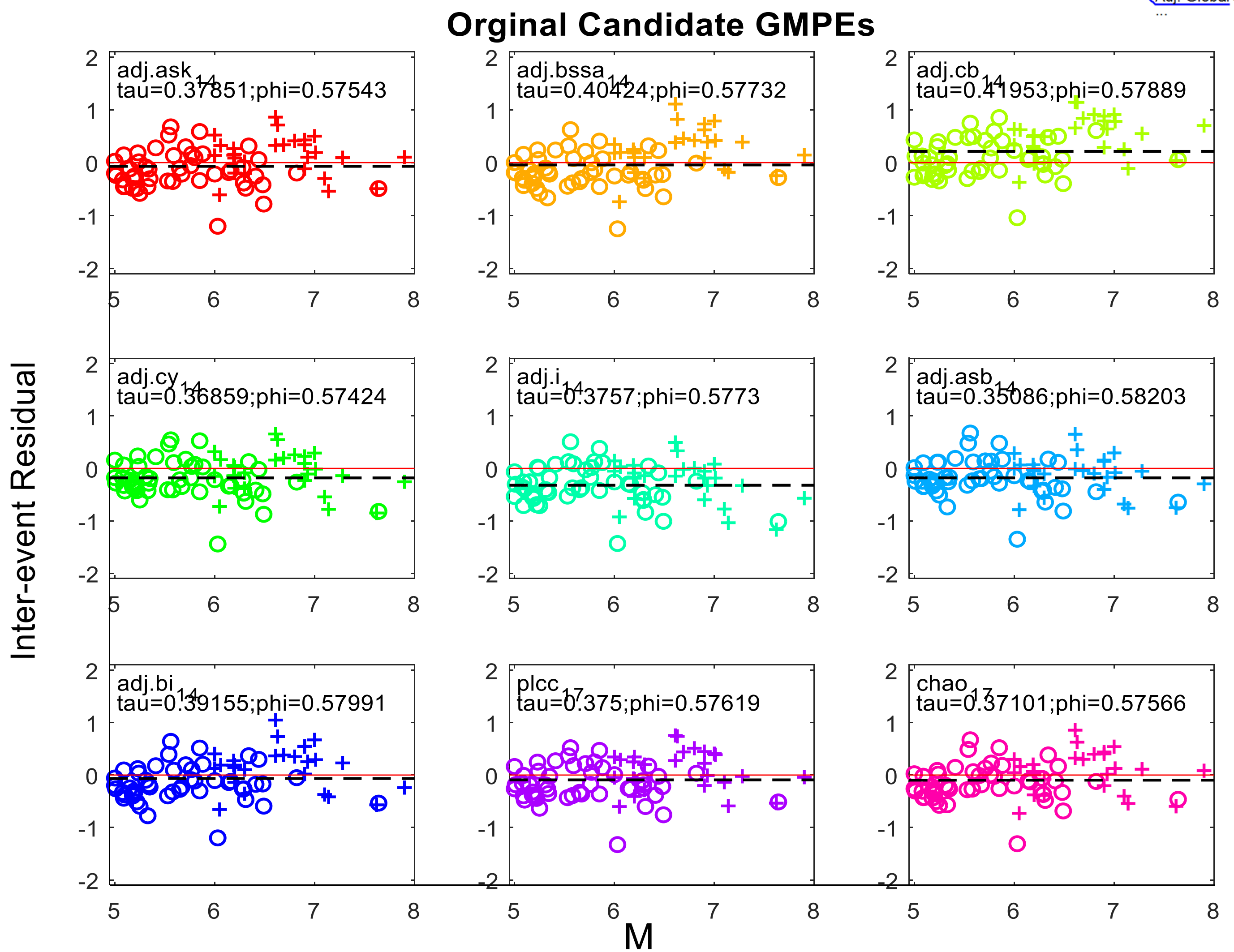
$$\text{Correction Factor} = \frac{GMPE(M,R,Vs30,F_{type})}{GMPE(M,R,Vs30=760,F_{type}=0(SS))}$$





Examine Candidate Models with respect to corrected data





Calculate mean between event residual and the Log-Likelihood

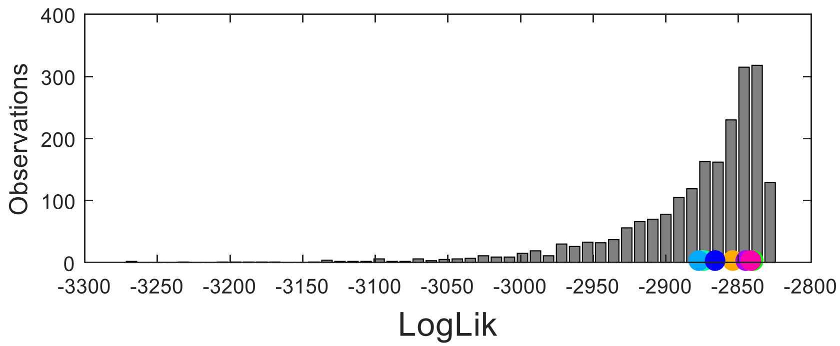
- Mixed effects models (Abrahamson and Youngs 1992):
  - $\ln y_{ij} = f\left(M_i, r_{ij}, \theta\right) + \eta_i + \varepsilon_{ij}$
- Based on the selected data and thousand GMPE models
  - Mean between event residuals.
  - Log-likelihood ($\tau = 0.38$ & $\phi = 0.58$)

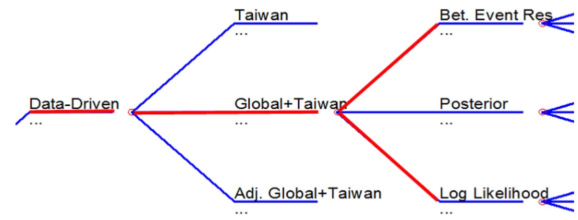
$$LnL = -\frac{N}{2}\ln(2\pi) - \frac{1}{2}\ln|C| - \frac{1}{2}(y-\mu)^T C^{-1}(y-\mu)$$

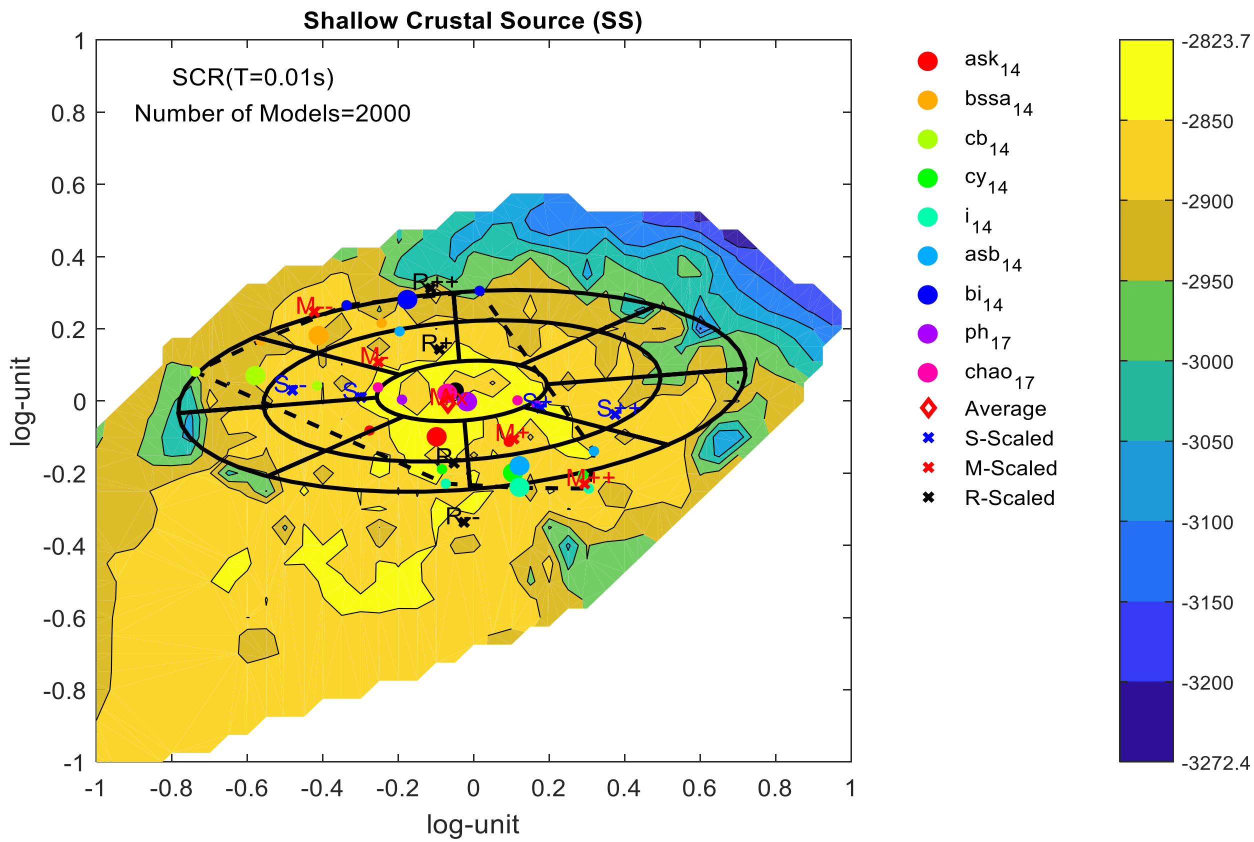
$$C = \begin{bmatrix} \sigma^2 I_{n1} + \tau^2 1_{n1} & 0 & \cdots & 0 \\ 0 & \sigma^2 I_{n2} + \tau^2 1_{n2} & \cdots & 0 \\ \vdots & \cdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma^2 I_{nM} + \tau^2 1_{nM} \end{bmatrix}$$

From Eq.7 (Abrahamson and Youngs 1992)

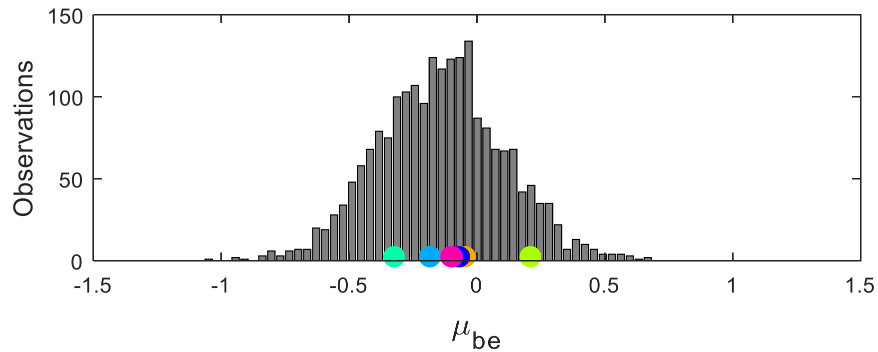
The Log-Likelihood contour plot

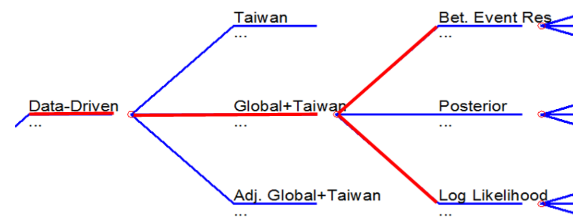


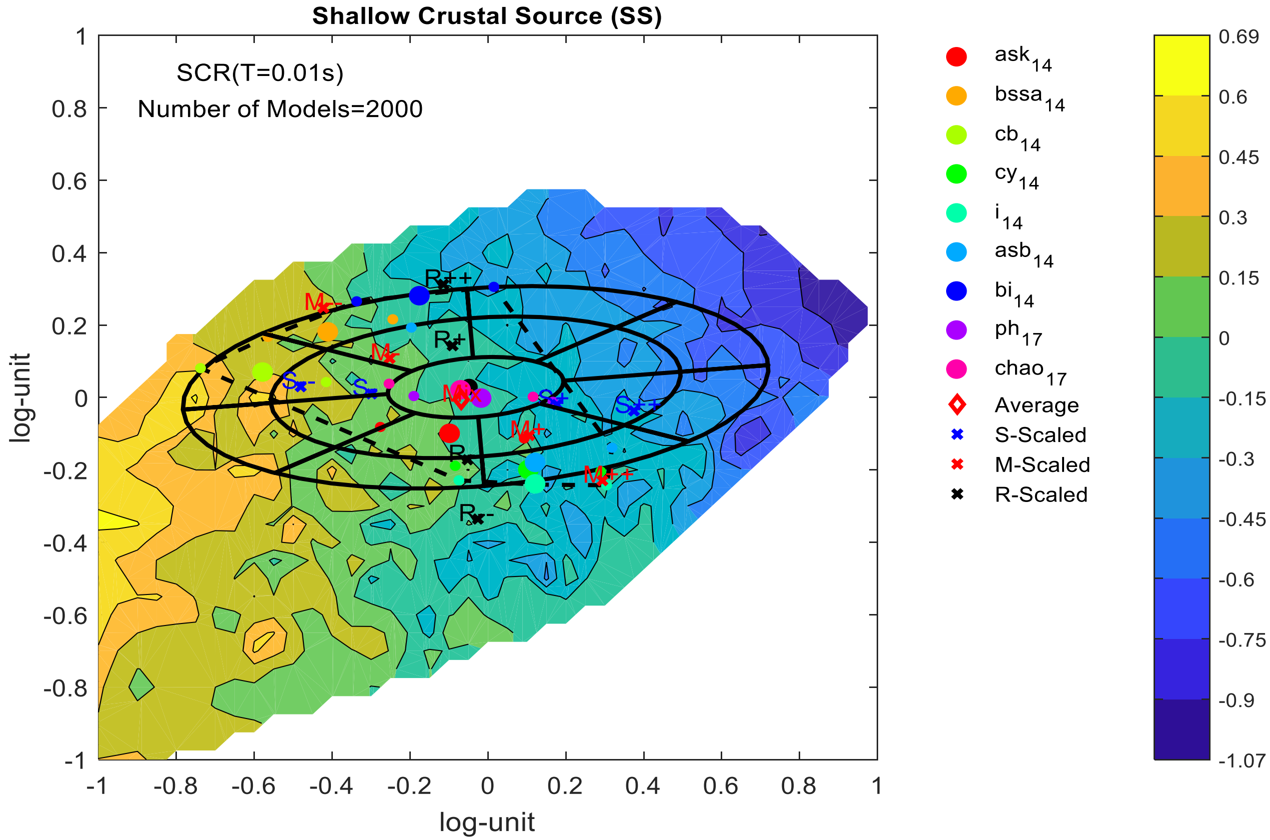




The Mean Between Event Residuals contour plot







Calculate the Weights

- According to SWUS report:

$$w_i = A_i \frac{1}{N_i} \sum_{j=1}^{N_i} L_{ji}$$

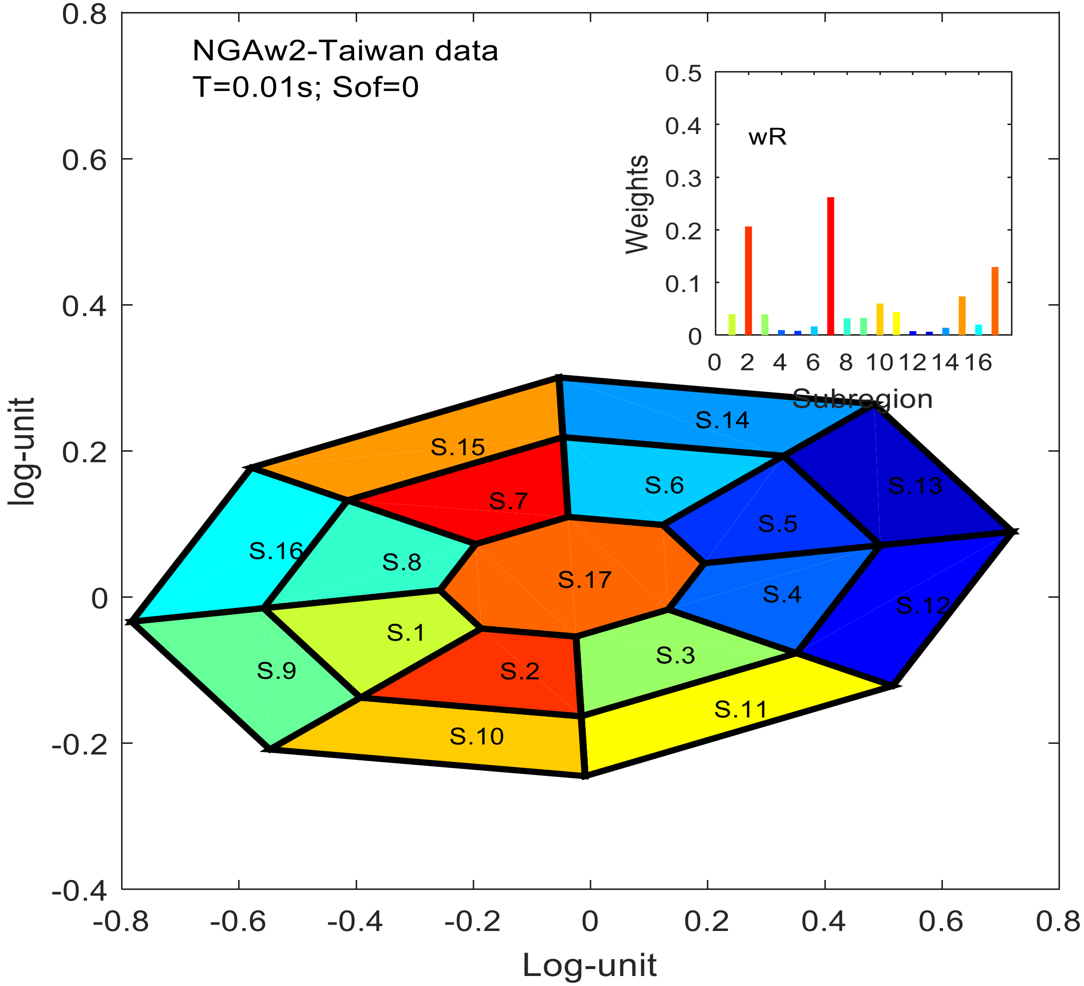
L_{ji} could be one of the following alternative metrics:

- $\frac{1}{|\mu(\delta Be)+c|}$, and c = 0.0075 (SWUS report)
- LogLik, (the likelihood);
- P, the "prior", which is the value of the probability density function of the coefficient distribution for each model.
- "Posterior", which is the prior times the likelihood.

Residual Weights ~ wR

$$w_i = A_i \frac{1}{N_i} \sum_{j=1}^{N_i} L_{ji}$$

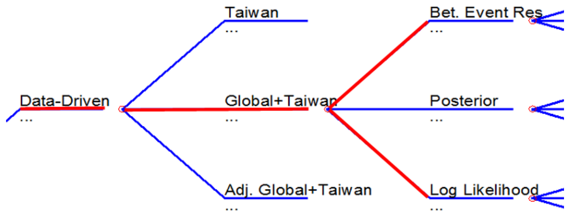
$$L_{ij} = \frac{1}{|\mu(\delta Be) + c|}, \qquad c = 0.0075$$

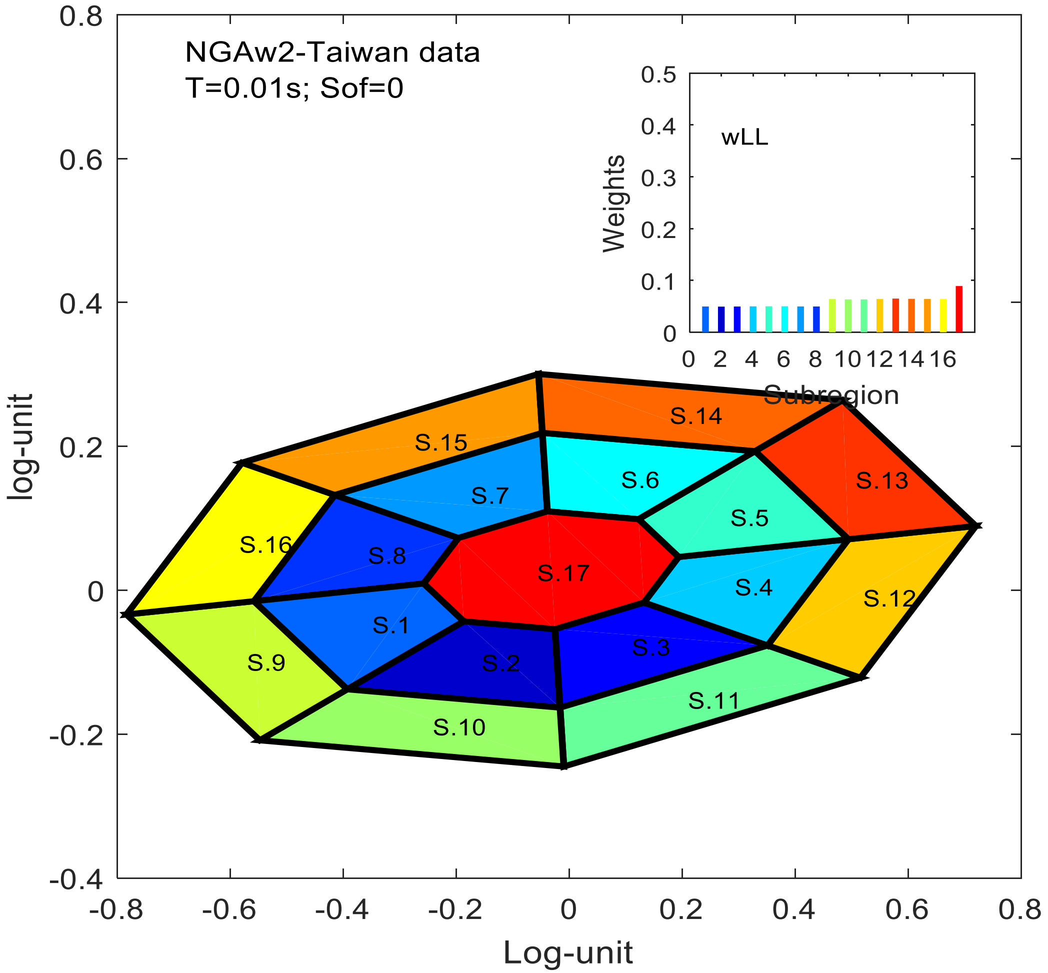


Log-likelihood Weights~ wLL

$$w_i = A_i \frac{1}{N_i} \sum_{j=1}^{N_i} L_{ji}$$

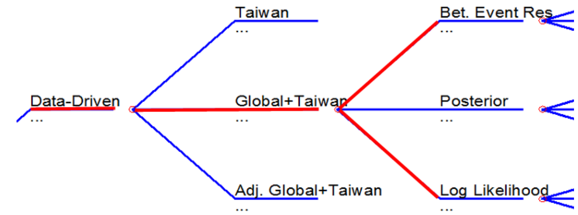
$$L_{ij} = log - likelihood$$

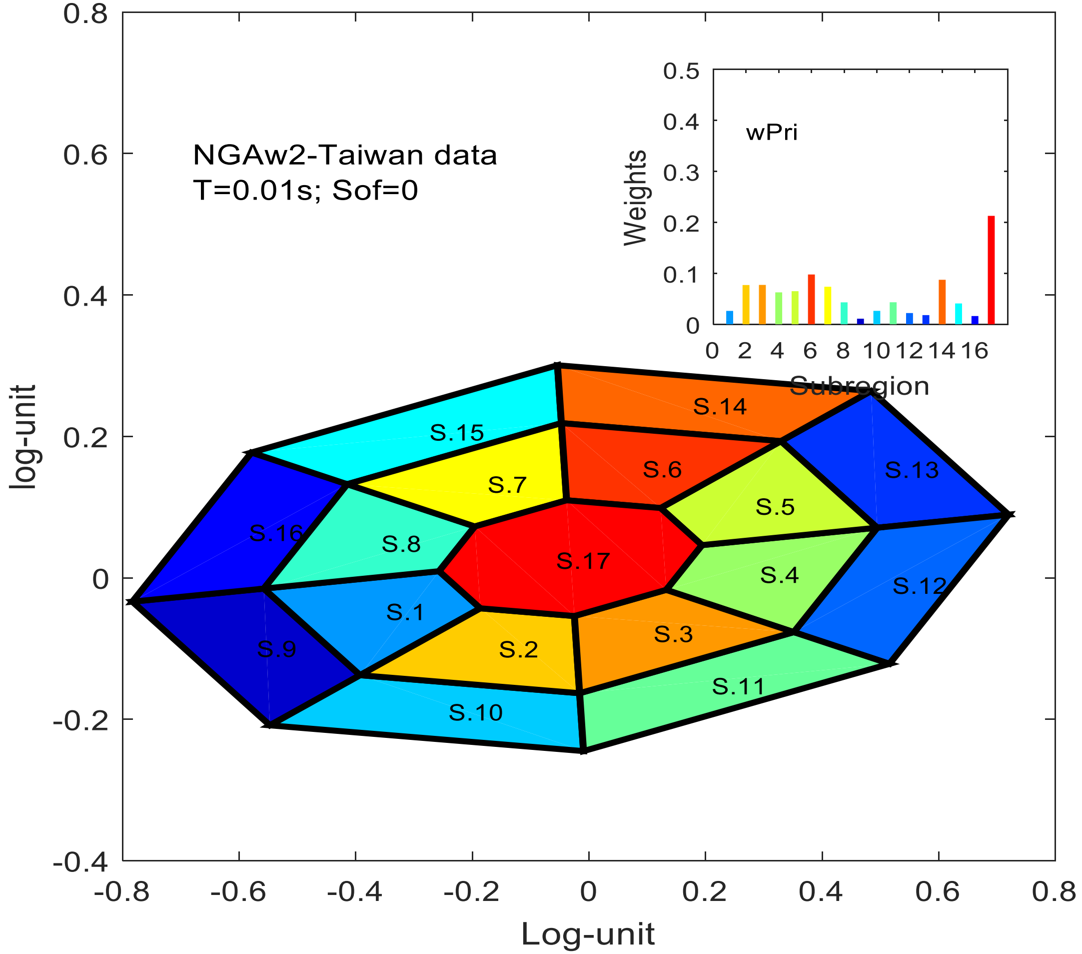




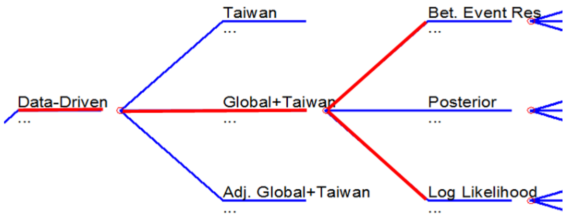
Prior Weights (wPri)

$$w_i = \frac{\sum_i^{Ni} pdf_i(x_i,\mu_\theta,\Sigma_\theta)}{pdf_0(x_0,\mu_\theta,\Sigma_\theta)}$$

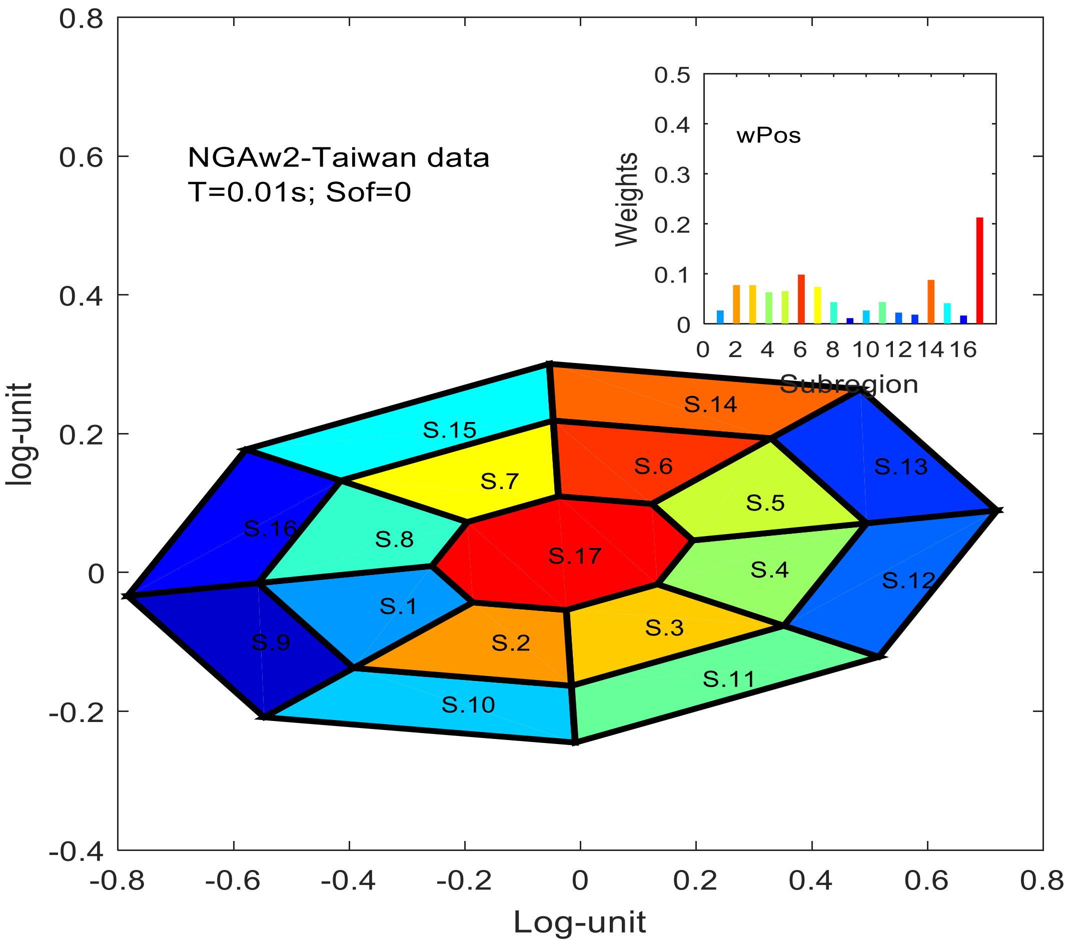




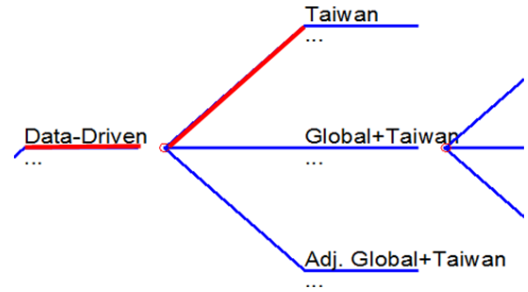
Posterior Weights (wPros)



$$w_i = \frac{\sum_i^{Ni} loglik_i * pdf_i(x_i, \mu_\theta, \Sigma_\theta)}{LogLik_0 * pdf_0(x_0, \mu_\theta, \Sigma_\theta)}$$



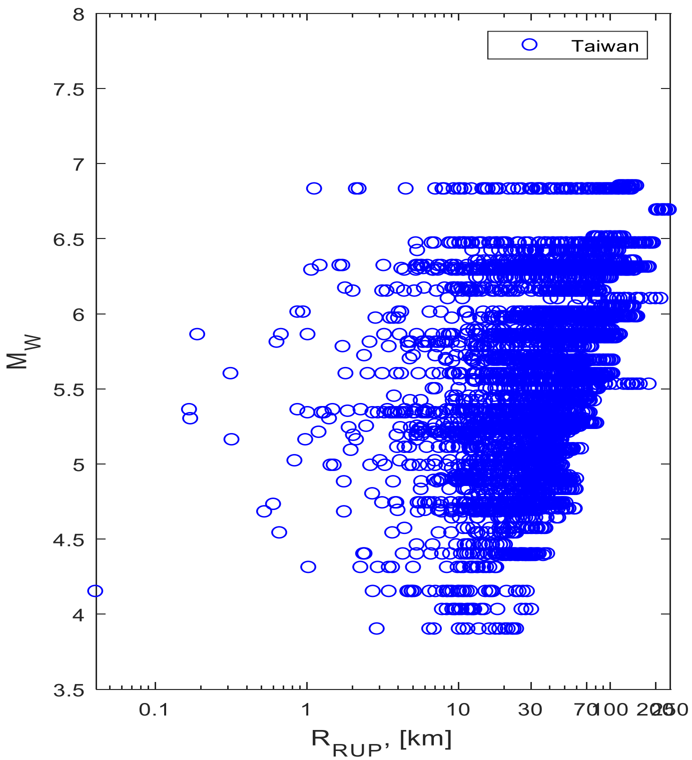
Taiwan data



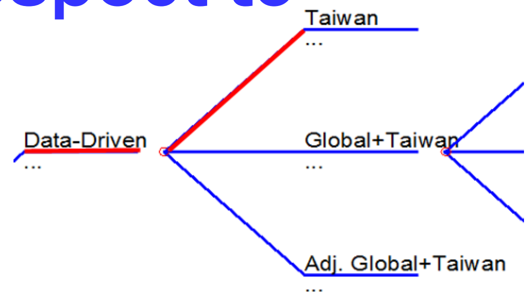
Data set used for adjusted GMPE models

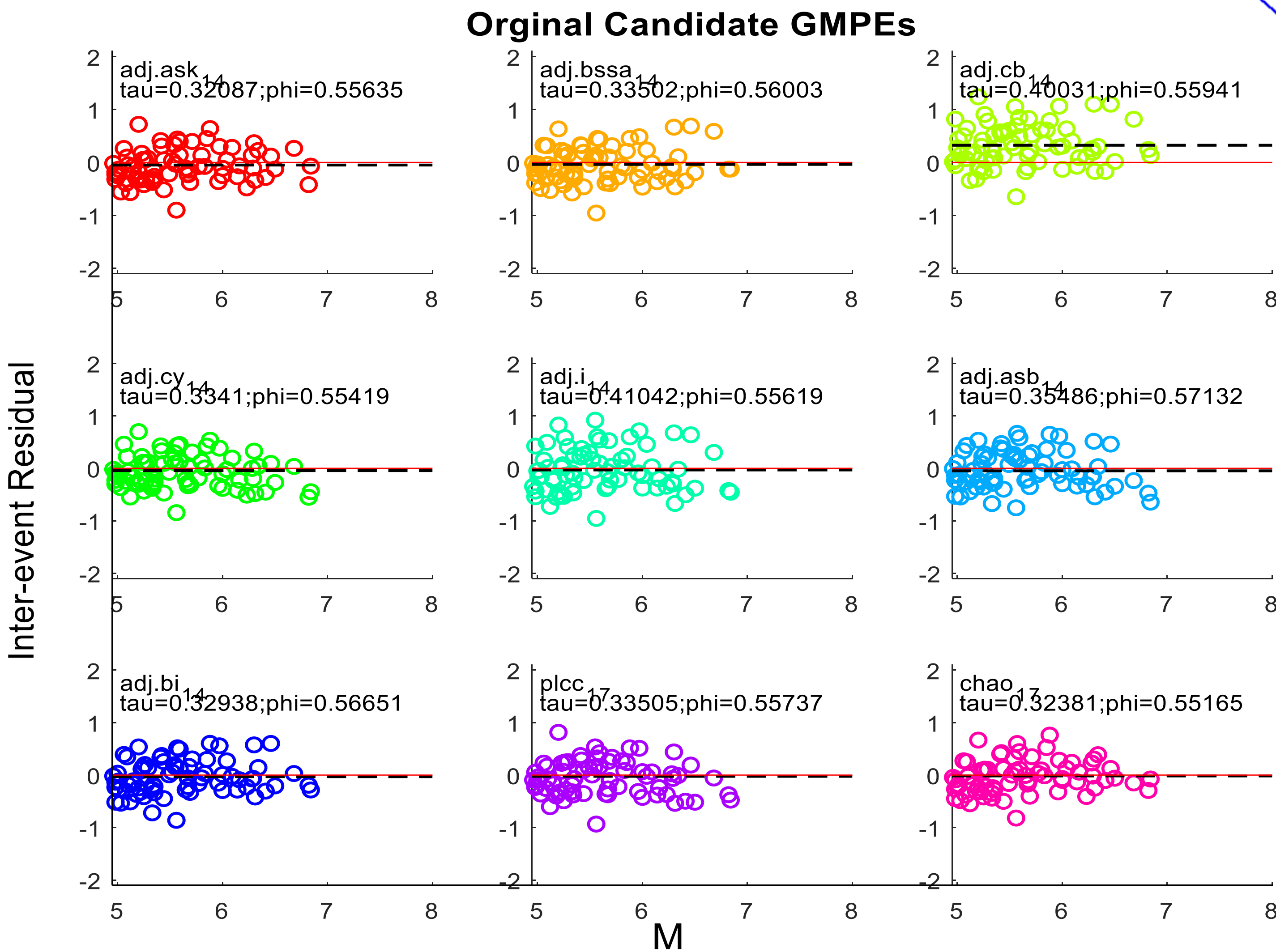
Selection criteria:

- Strike Slip, Reverse, and Normal
- Mw≤ 7.0
- $R_{RUP} \leq Rmax$
- At least 15 records/events
- Remove aftershock events
- Remove Chi-Chi Mw7.65

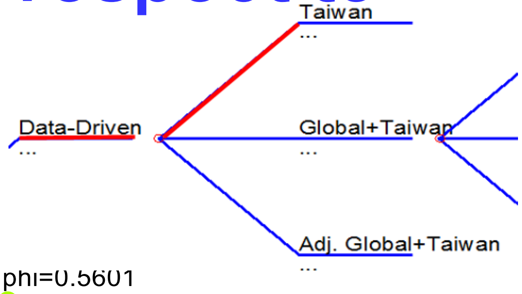


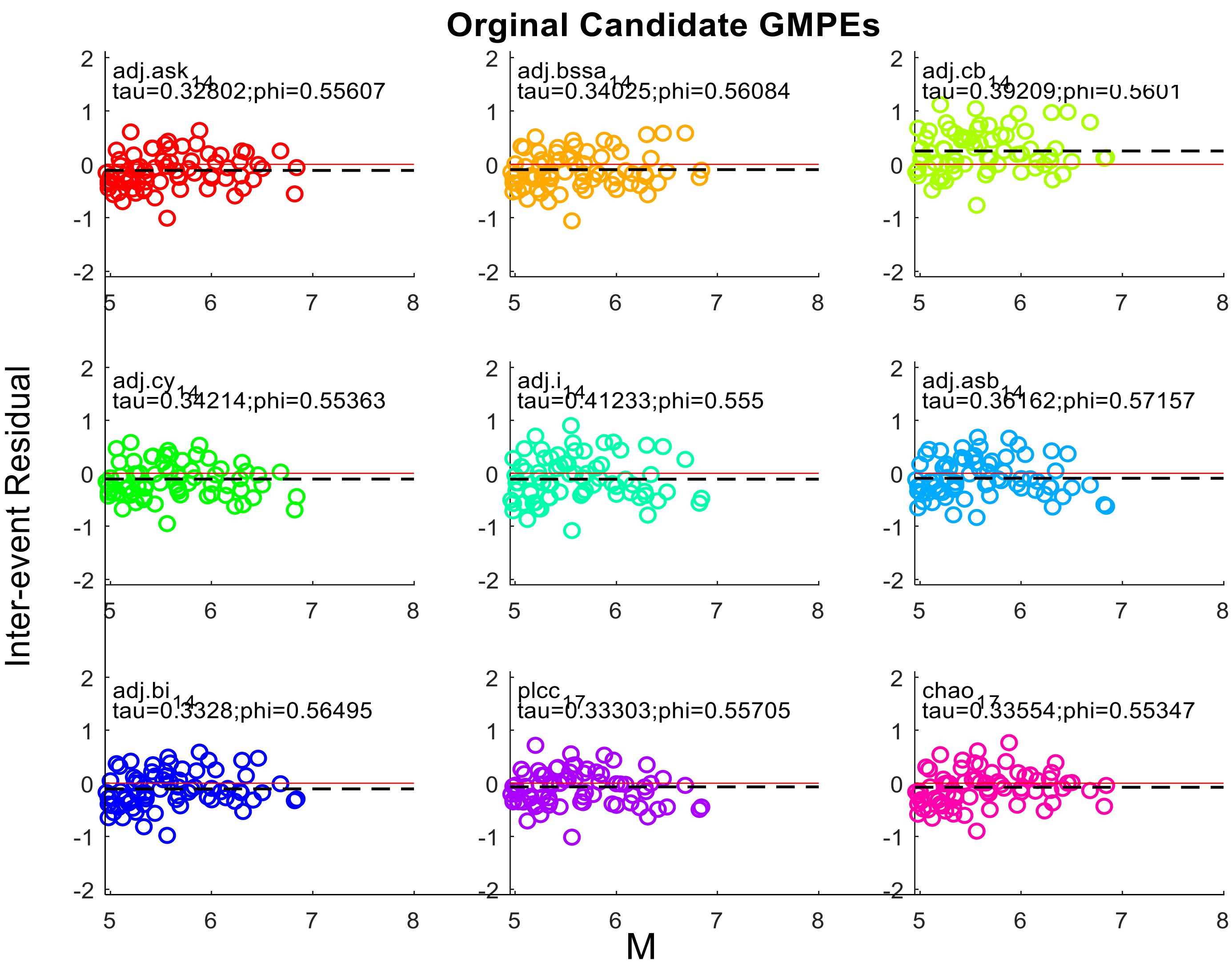
Examine Candidate Models with respect to truly recorded data



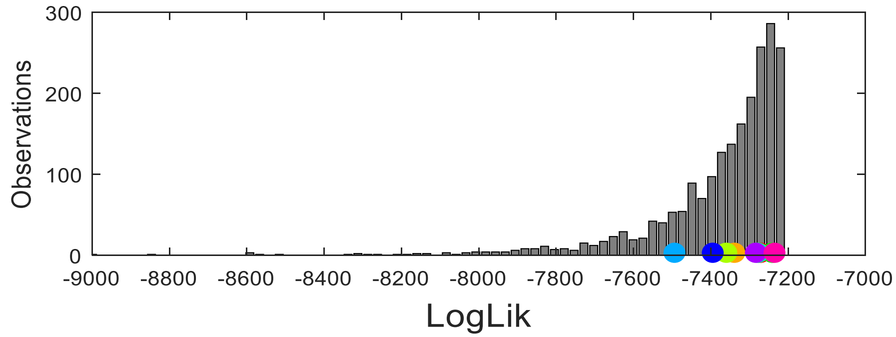


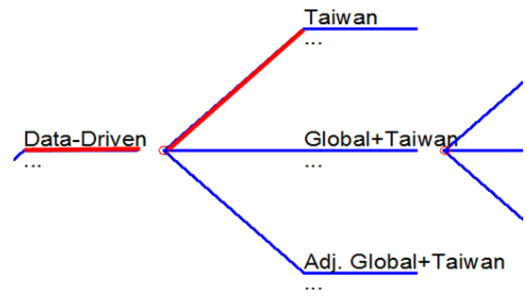
Examine Candidate Models with respect to corrected data

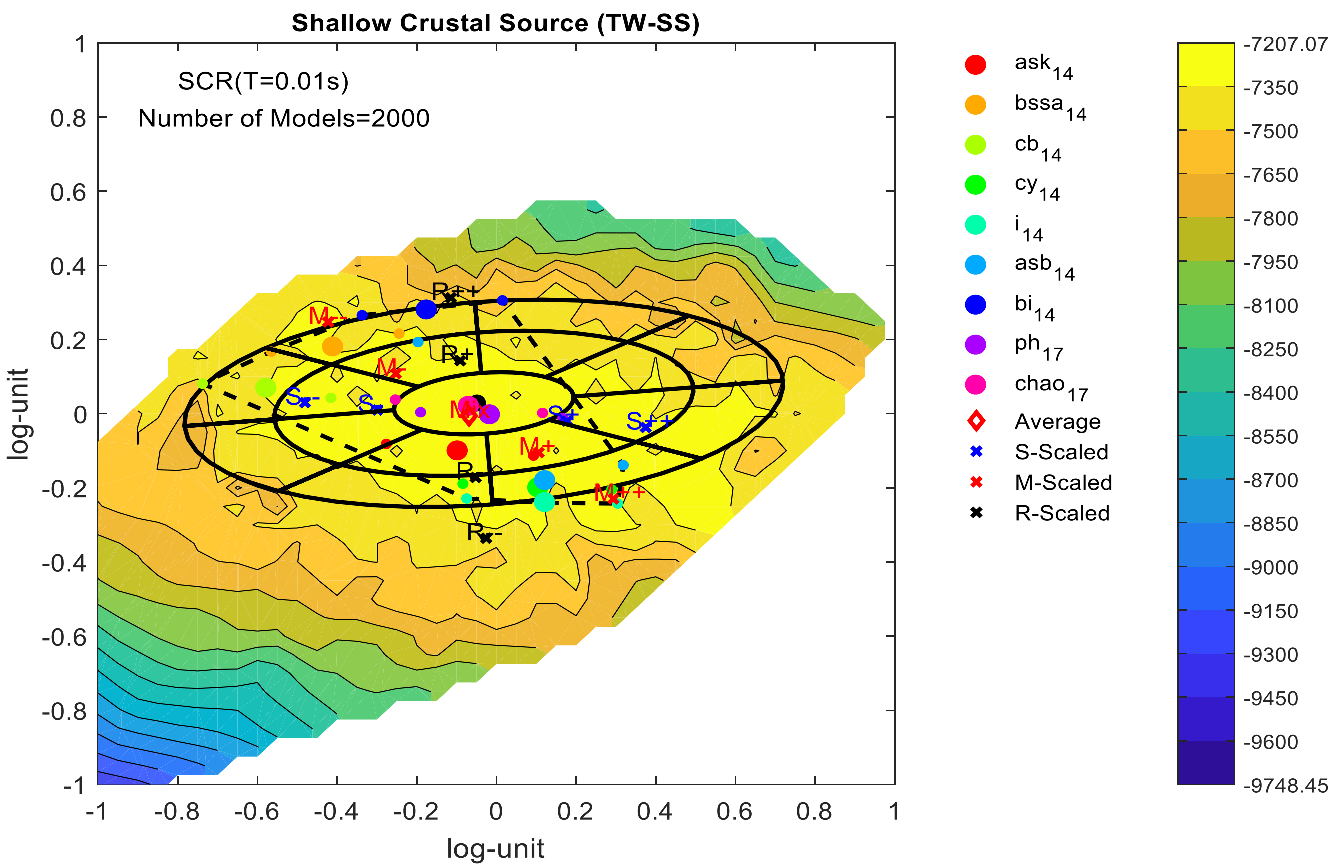




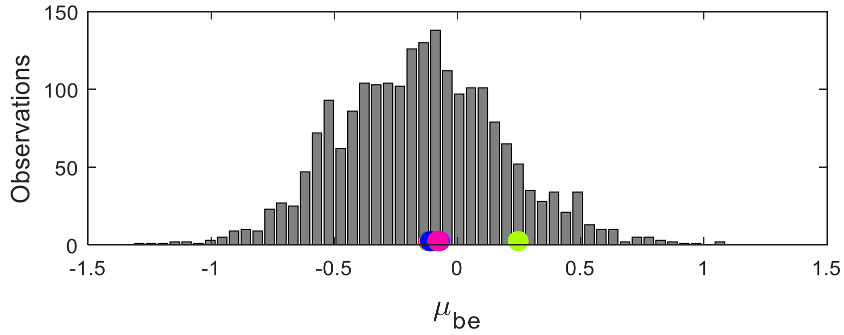
The Log-Likelihood contour plot-TW

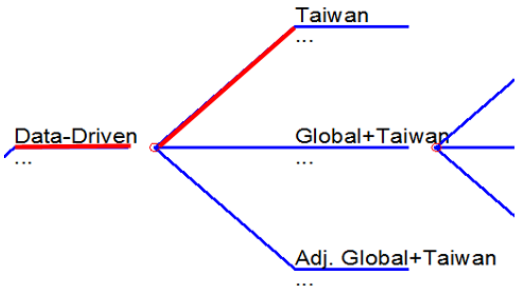


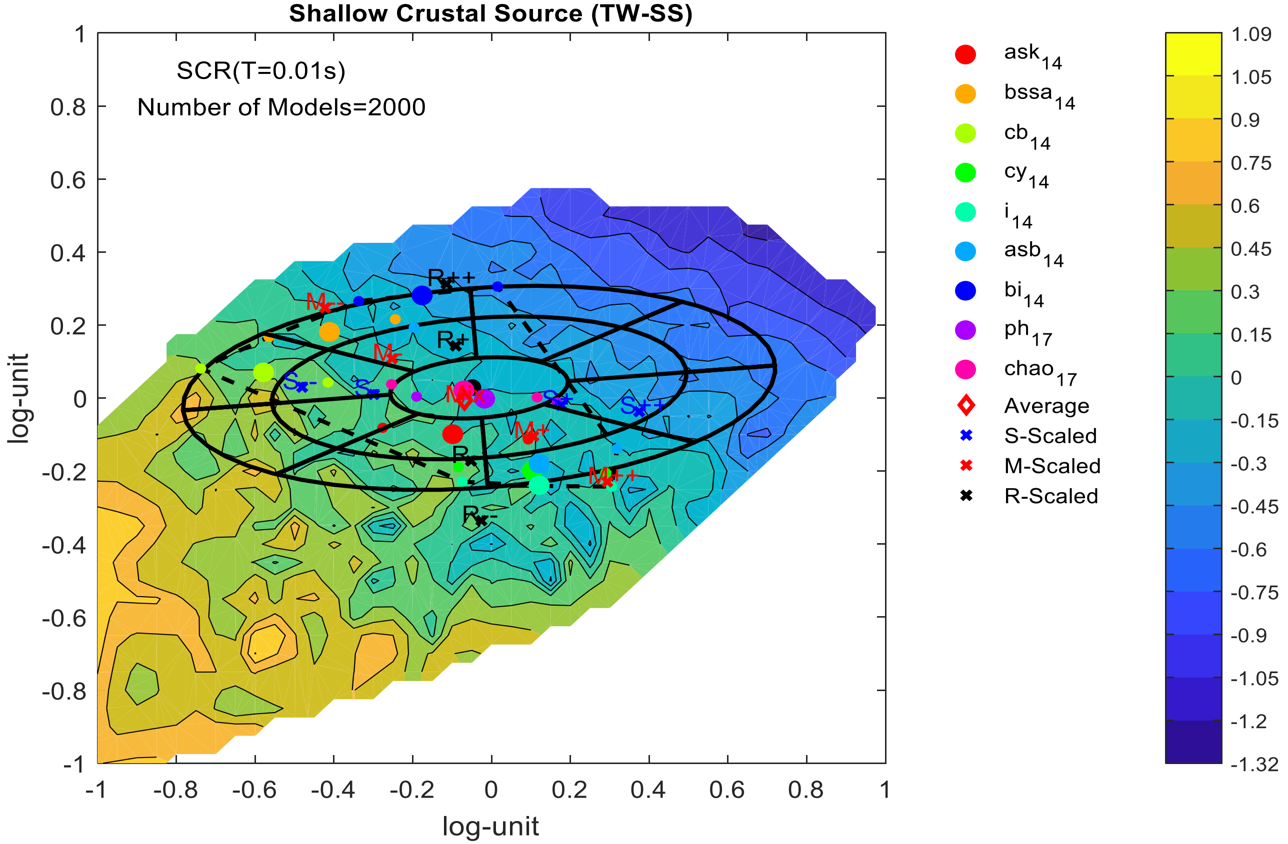




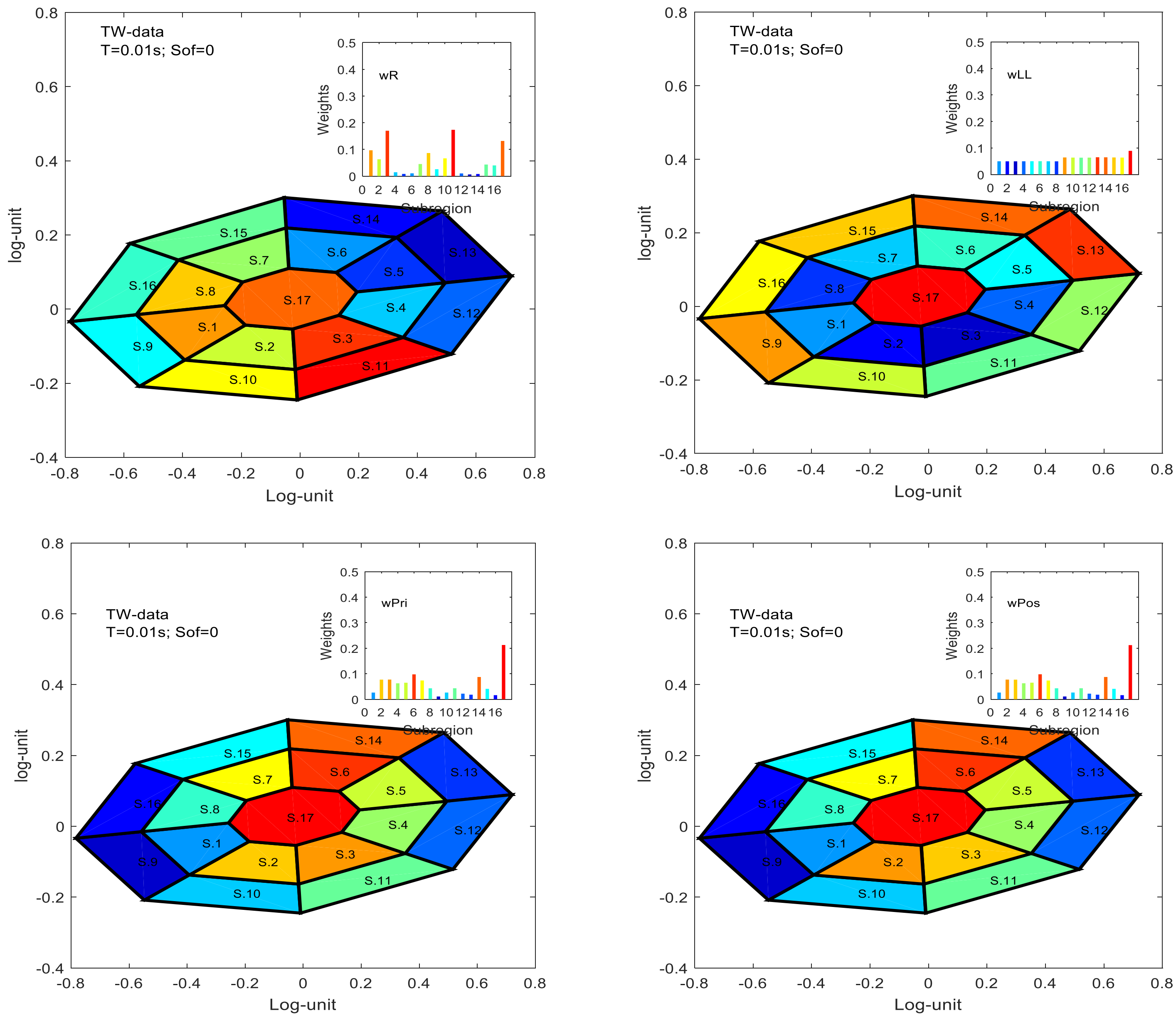
The Mean Between Event Residual contour plot-TW



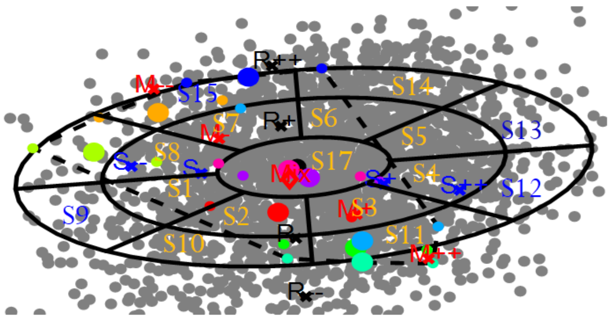


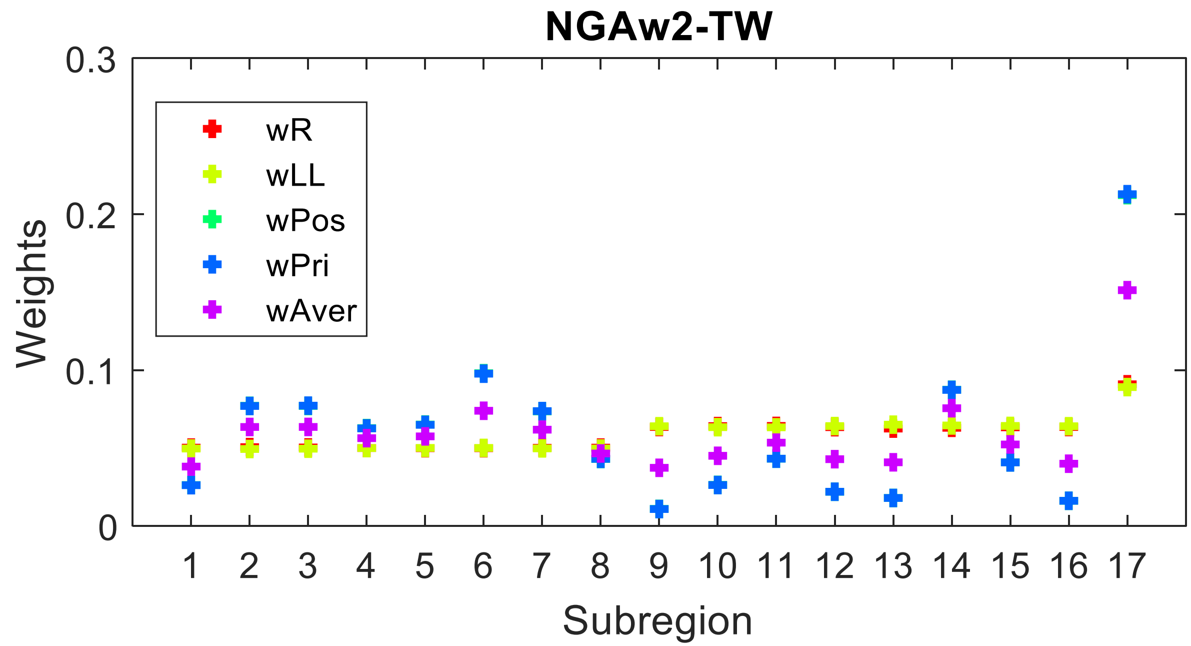


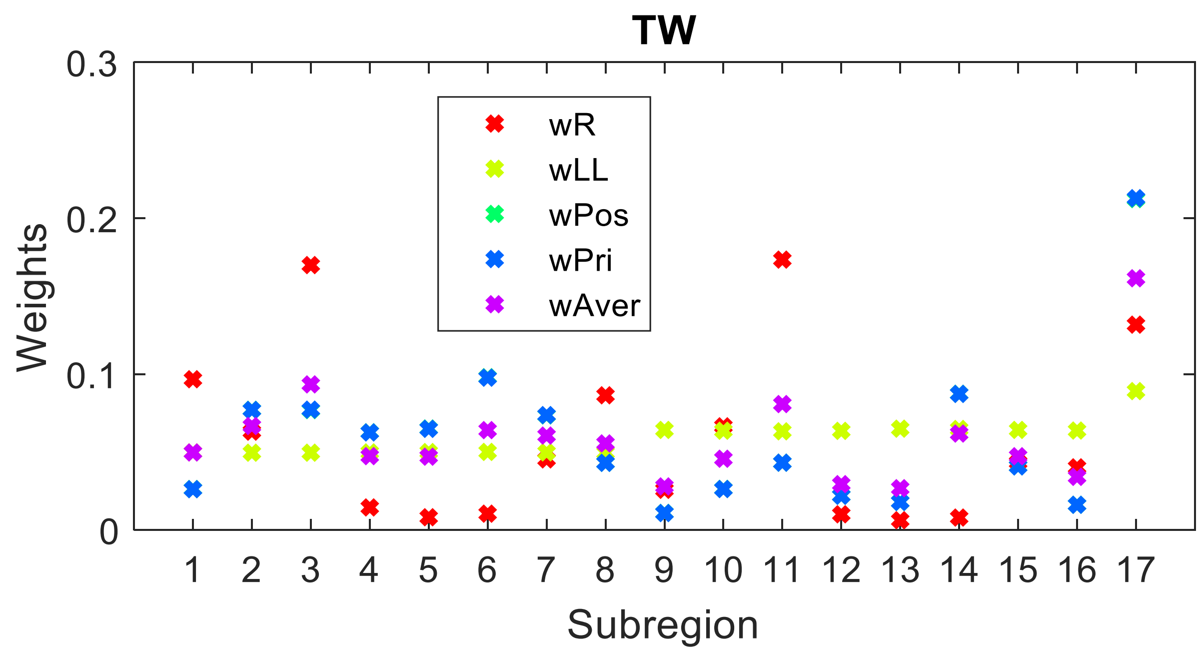
Weights Scheme



Weights Scheme







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Thank you for your attention !

Questions ?

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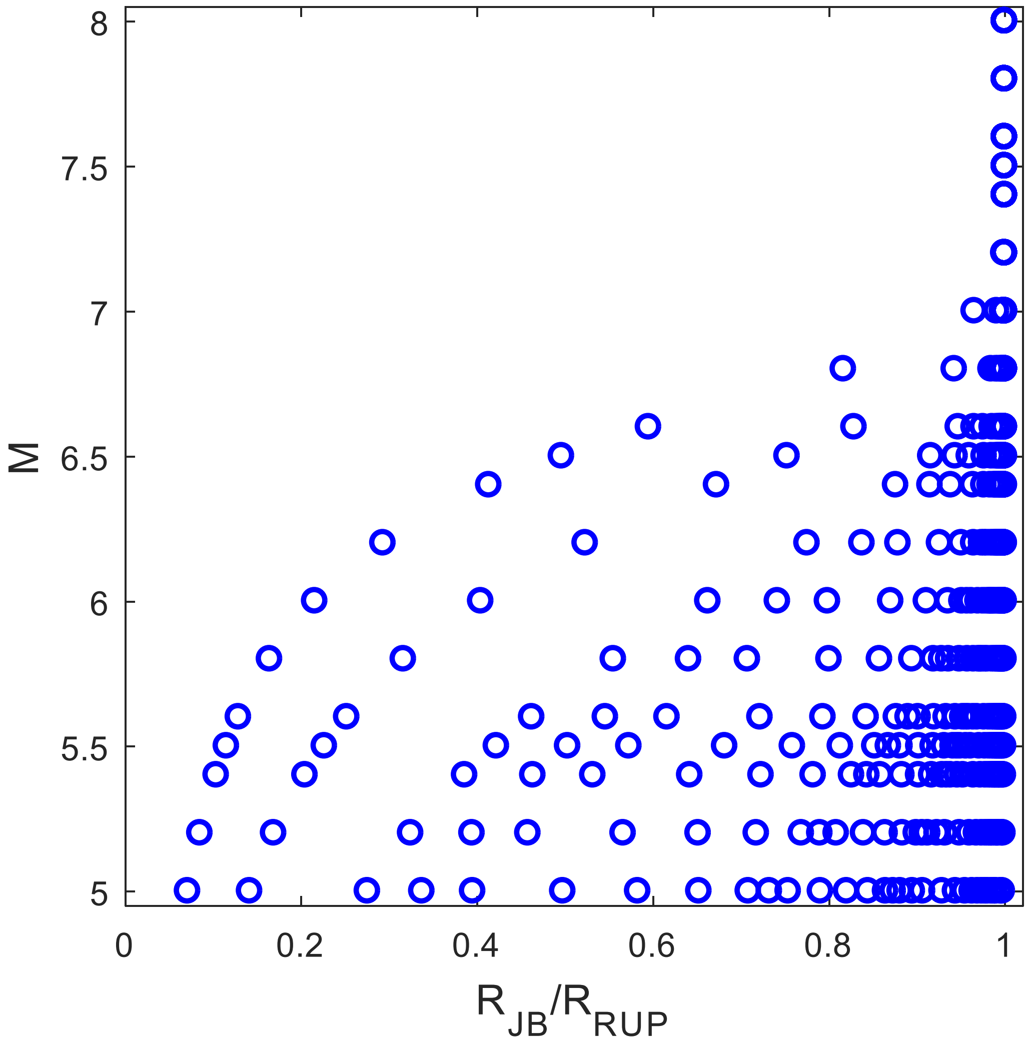

8
7.5
7
6.5
6
5.5
5
M
0
0.2
0.4
0.6
0.8
1
R_{JB}/R_{RUP}